ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Dynamic 2D/3D Registration

Sofien Bouaziz Andrea Tagliasacchi Mark Pauly

Presenters



Sofien Bouaziz



Andrea Tagliasacchi



Mark Pauly



Presenters



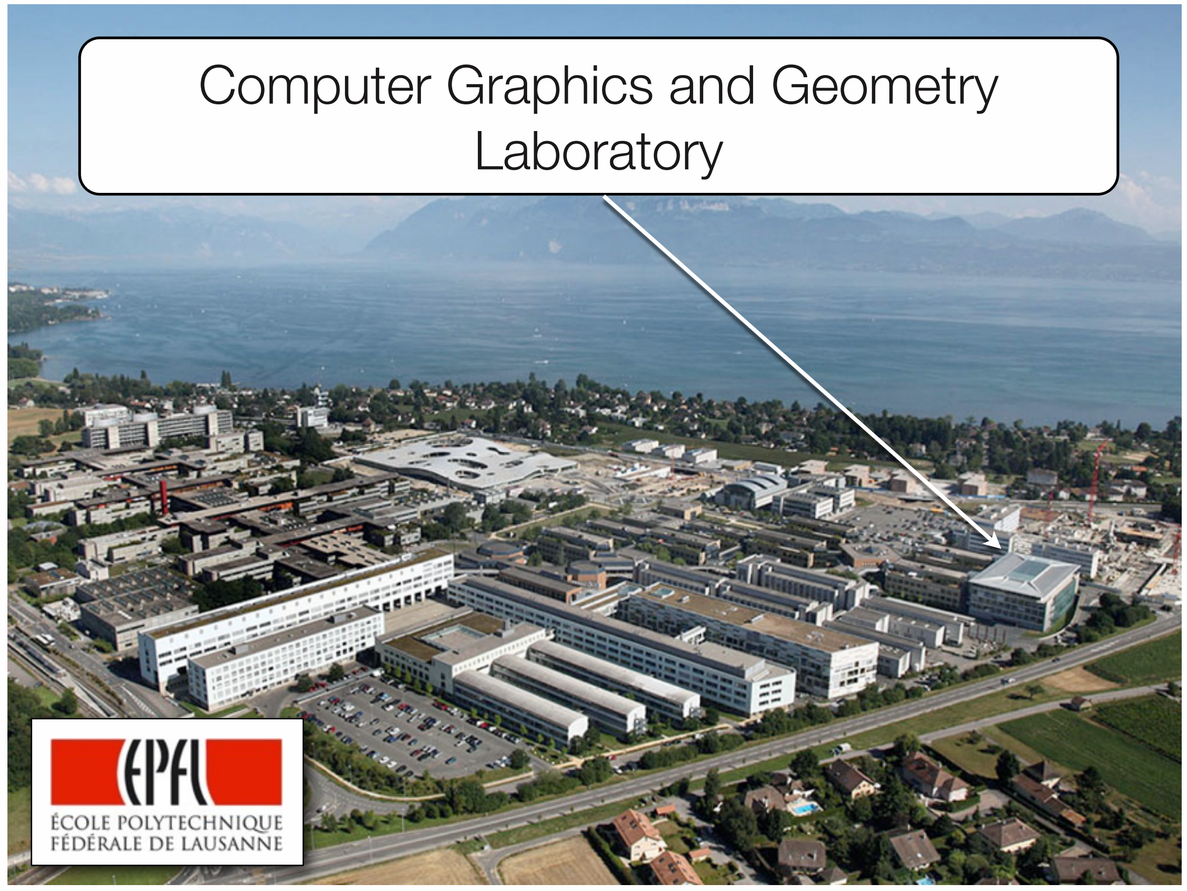
Sofien Bouaziz



Andrea Tagliasacchi



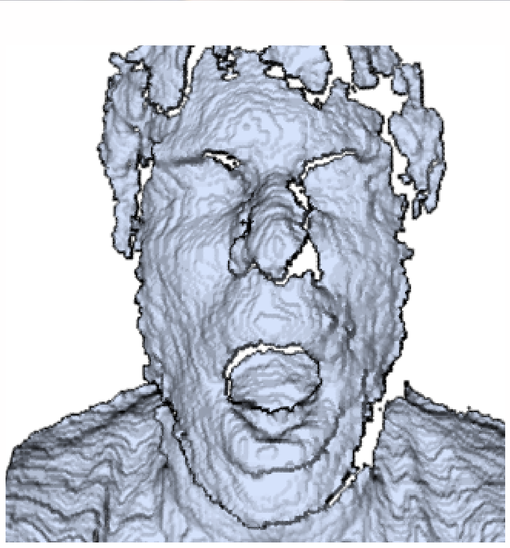
Mark Pauly



RGB-D Sensors

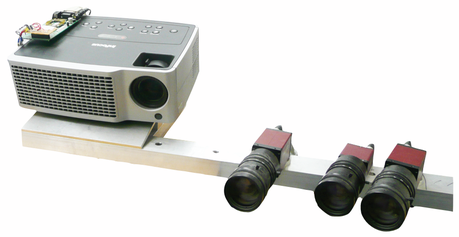






RGB-D Sensors

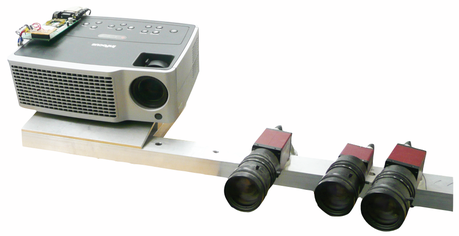
2008



RGB-D Sensors



2008

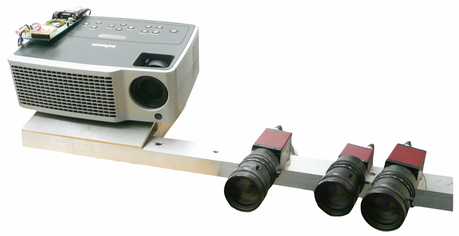


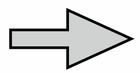
RGB-D Sensors



2008

2010







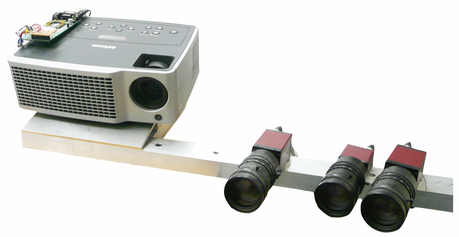
RGB-D Sensors



2008



2010





RGB-D Sensors

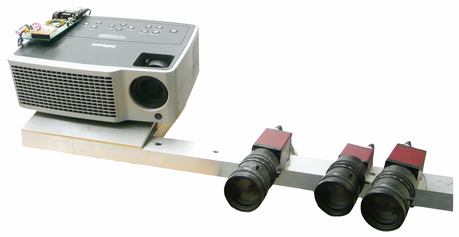


2008



2010

2013







RGB-D Sensors

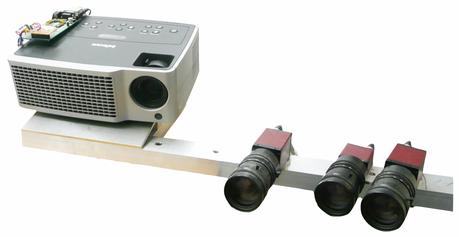


2008



2010

2013











RGB-D Sensors

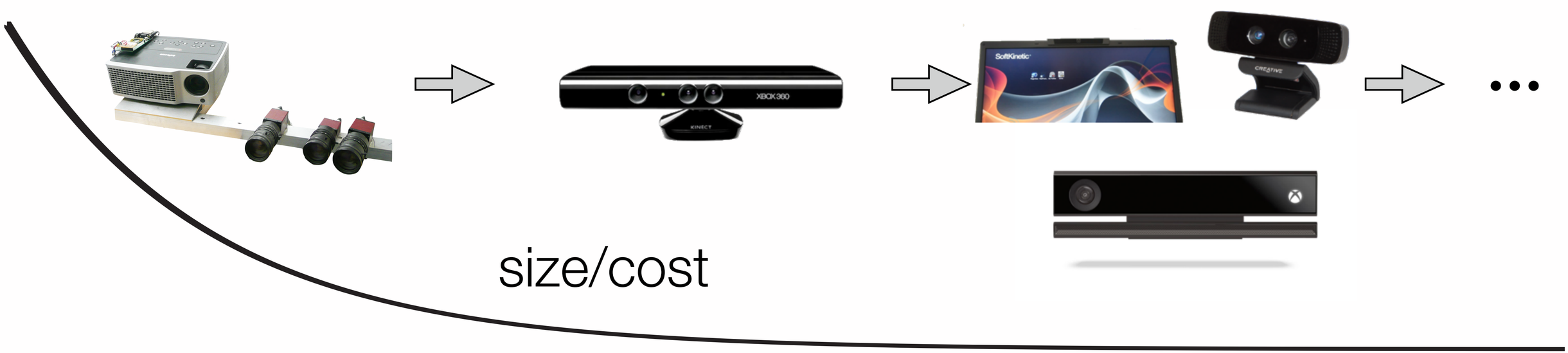


2008



2010

2013

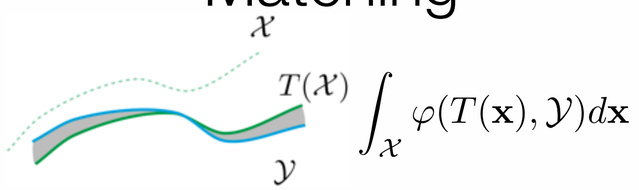


Outline

Theory

$$\arg\min_{T} \quad E_{\text{match}} + \sum_i w_i E^i_{\text{prior}}$$

Matching



$$\int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

Regularizing

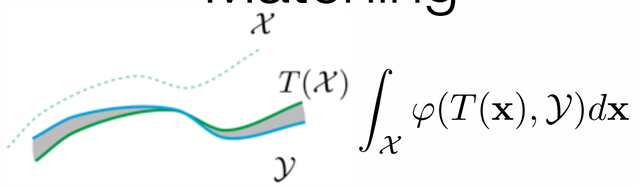


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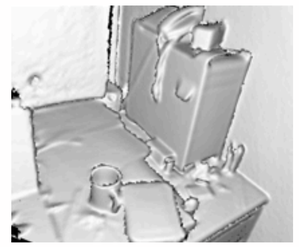
$$\int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

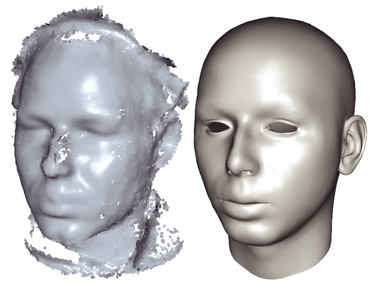
Regularizing



Applications







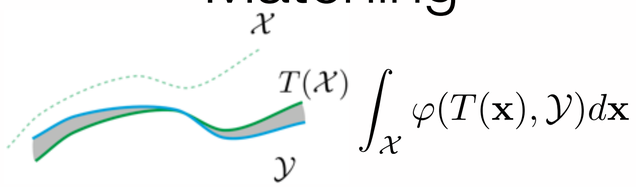


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$$\arg\min_{T} \quad E_{\text{match}} + \sum_i w_i E^i_{\text{prior}}$$

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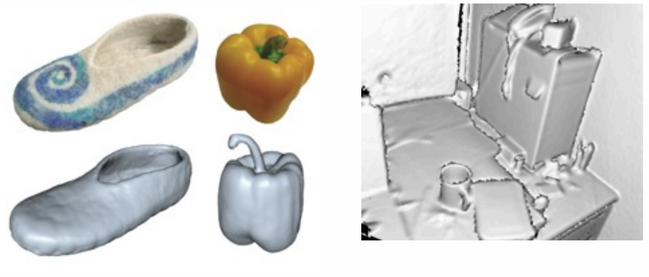
Demos and code

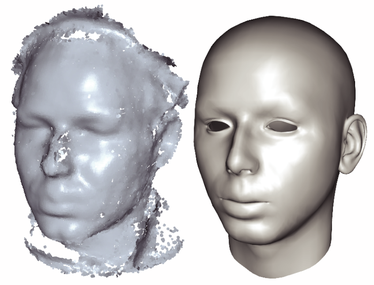
faceshift

pcl

MATLAB

Applications







Overview

Introduction (10min)

- Registration (10 min)
  - 3D Geometry (30 min)
  - 2D Images (20 min)
  - Combined 2D/3D (5 min)
  - Robust Registration (20 min)
  - Q&A (5min)
- Applications
  - Rigid Scanning (25 min)
  - Non-rigid Modeling (10 min)
  - Realtime Face Tracking (5 min)

Conclusions (5 min)

Q&A (5 min)


faceshift
Aisle
800
Motion Capture


faceshift
Aisle
800
Motion Capture
DEMO

Overview

Introduction



- Registration
  - 3D Geometry
  - 2D Images
  - Combined 2D/3D
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  - Rigid Scanning
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Conclusions

Q&A

Registration - Examples

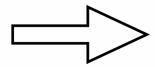
- Scan to scan

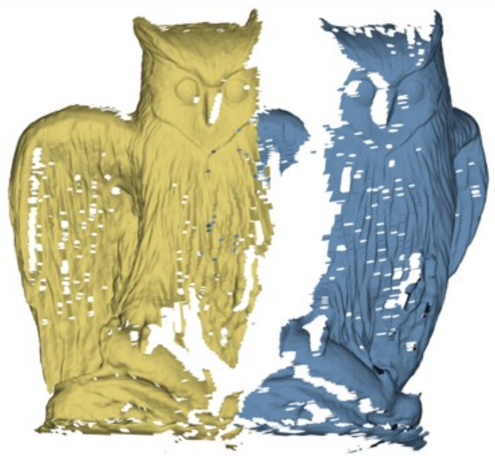


Registration - Examples

- Scan to scan

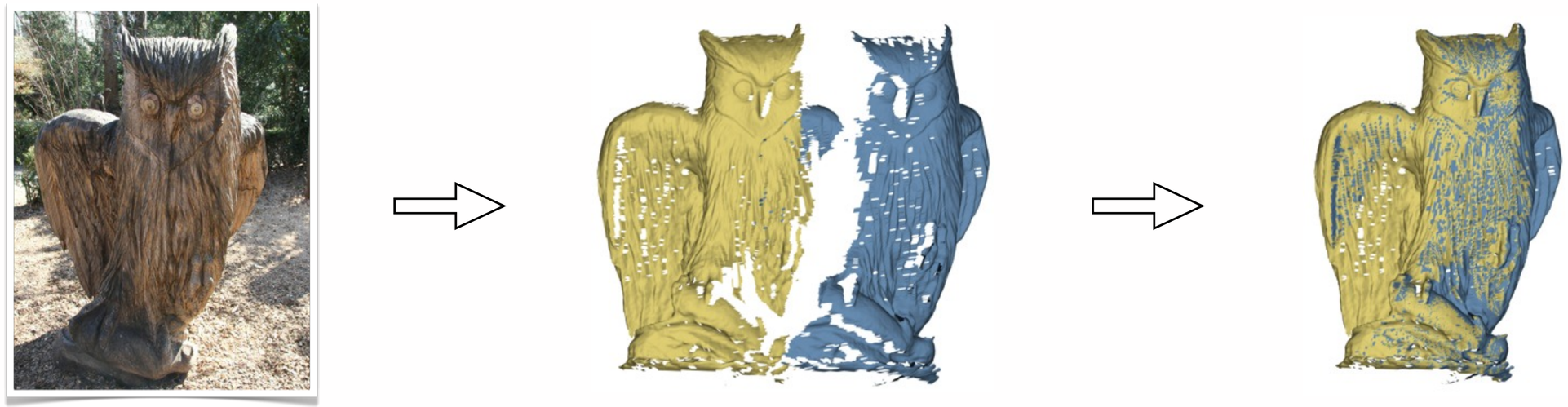






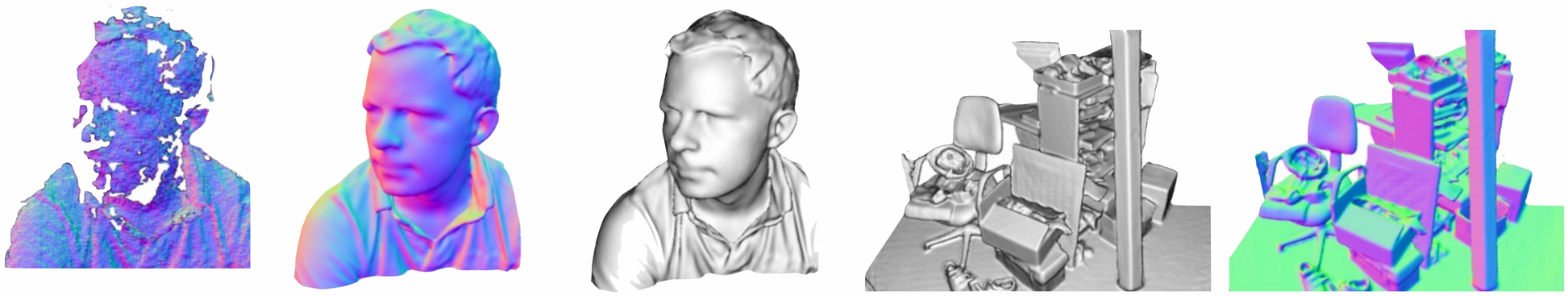
Registration - Examples

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Registration - Examples

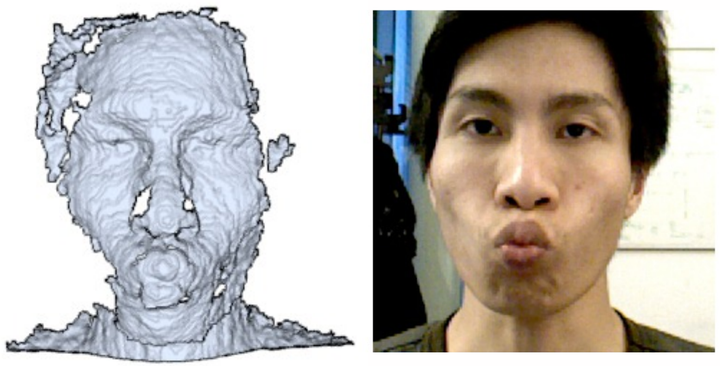
- Scan to scan



Newcombe, Izadi, Hilliges, Molyneaux, Kim, Davison, Kohli, Shotton, Hodges, Fitzgibbon:
KinectFusion: Real-Time Dense Surface Mapping and Tracking, *ISMAR 2011*

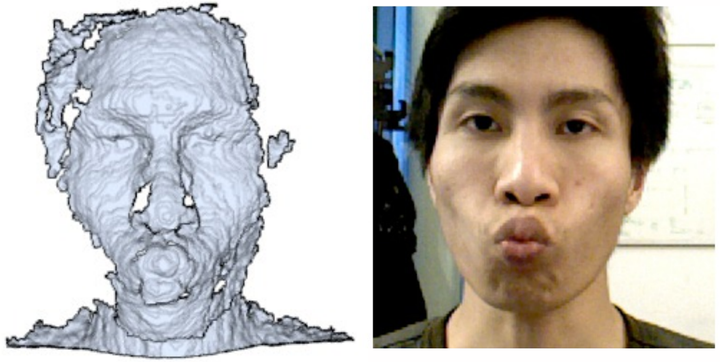
Registration - Examples

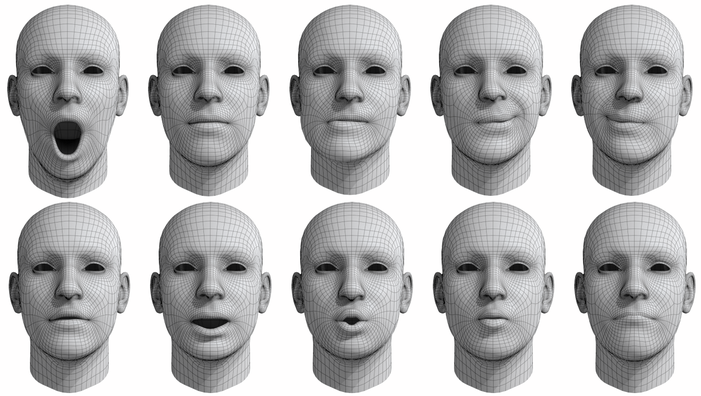
- Parameterized template to scan



Registration - Examples

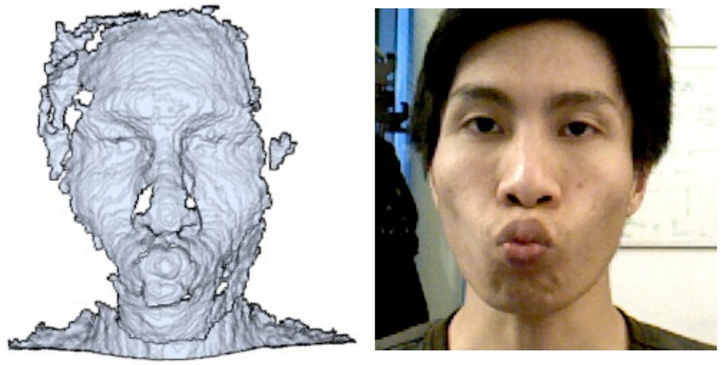
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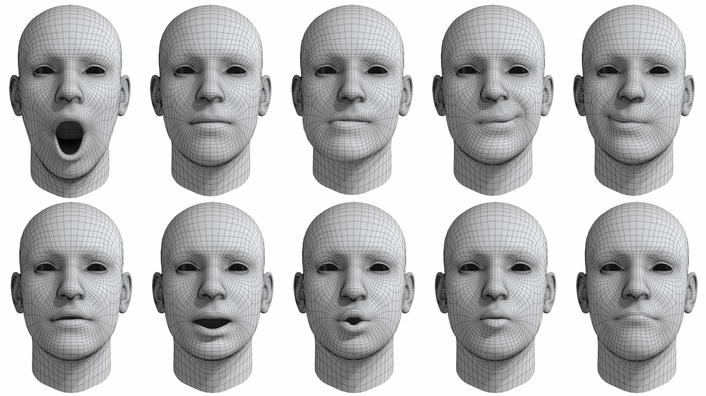


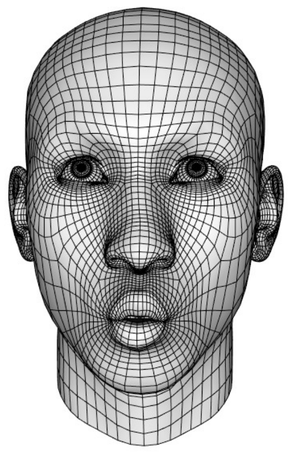


Registration - Examples

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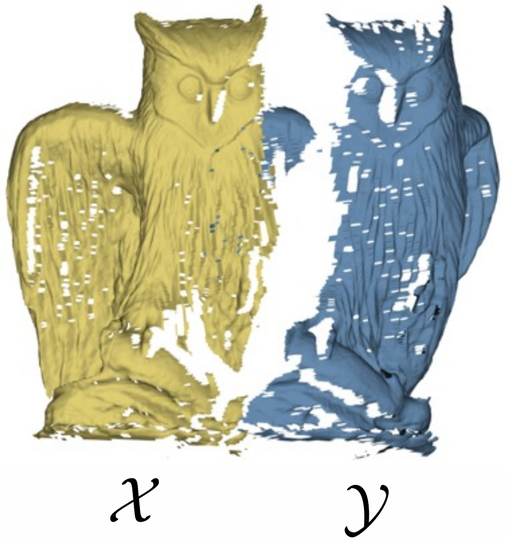






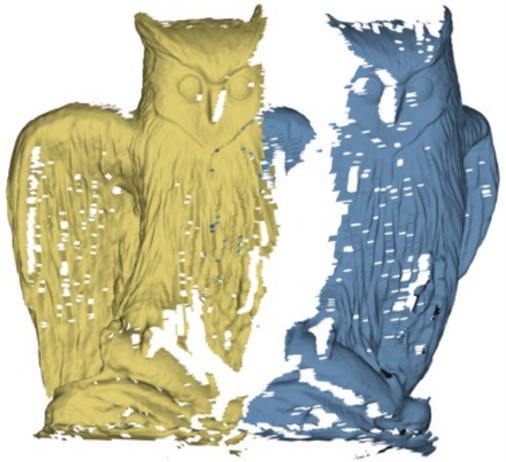
Pairwise Registration

- Align a *source model* $\mathcal{X}$ onto a *target model* $\mathcal{Y}$



Pairwise Registration

- Align a *source model* $\mathcal{X}$ onto a *target model* $\mathcal{Y}$
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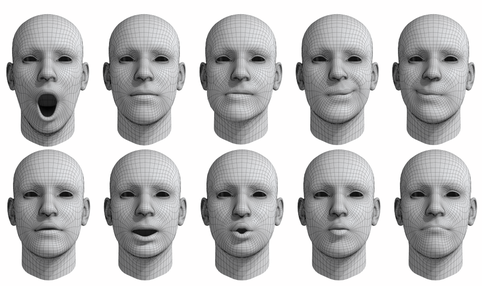
$\mathcal{X}$ $\mathcal{Y}$



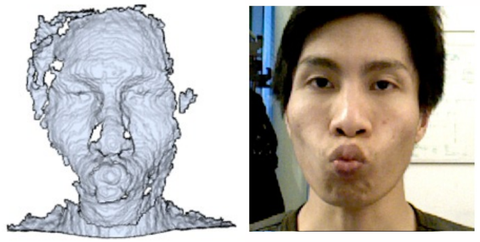
$\mathcal{Z} = T(\mathcal{X})$ $\mathcal{Y}$

Pairwise Registration

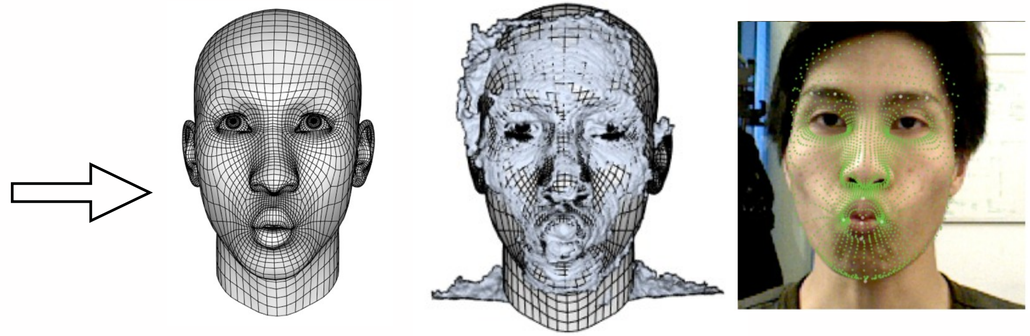
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$\mathcal{X}$



$\mathcal{Y}$



$\mathcal{Z} = T(\mathcal{X})$

Pairwise Registration

- Align a *source model* $\mathcal{X}$ onto a *target model* $\mathcal{Y}$
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- Two main questions:

Pairwise Registration

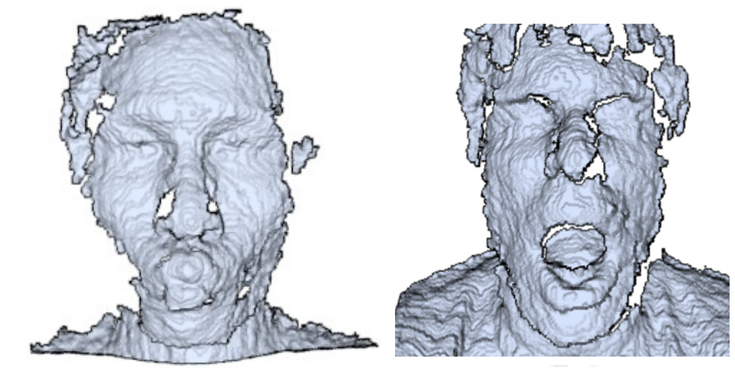
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- Two main questions:
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Pairwise Registration

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  - find a *transformation* $T(\mathcal{X})$ that brings $\mathcal{X}$ into alignment with $\mathcal{Y}$
- Two main questions:
  - How do we measure the quality of the alignment?
  - What transformations are allowed / good?

Pairwise Registration

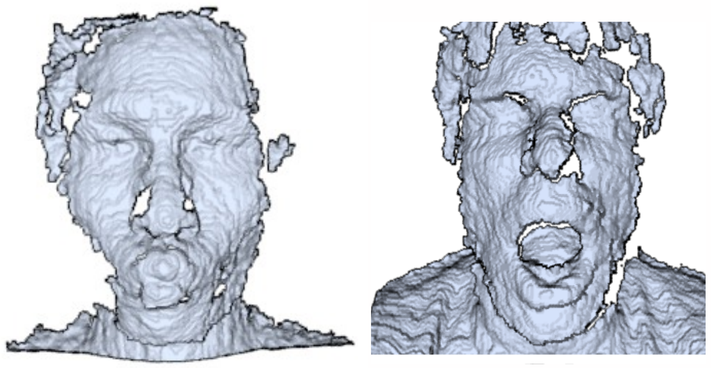
- Issues



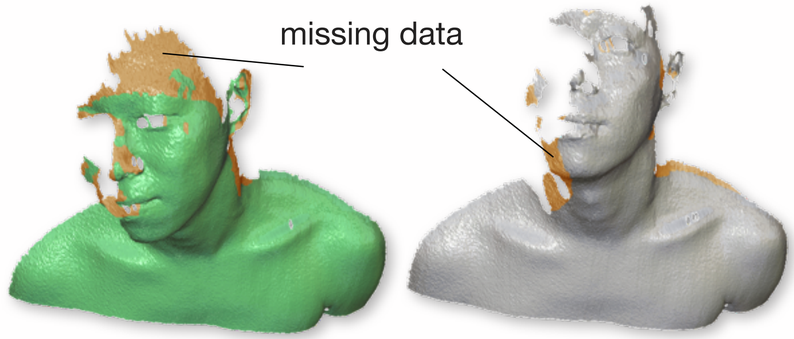
Noise

Pairwise Registration

- Issues



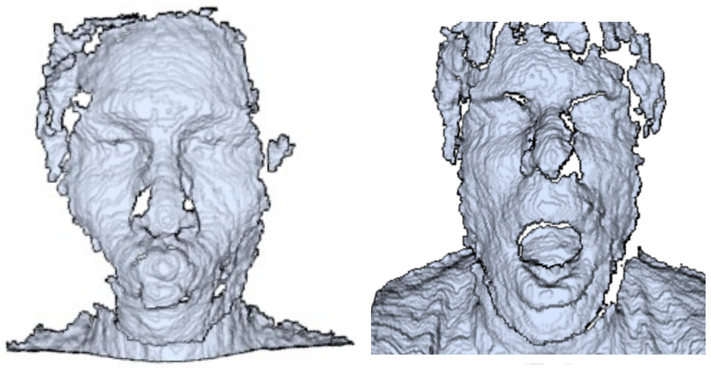
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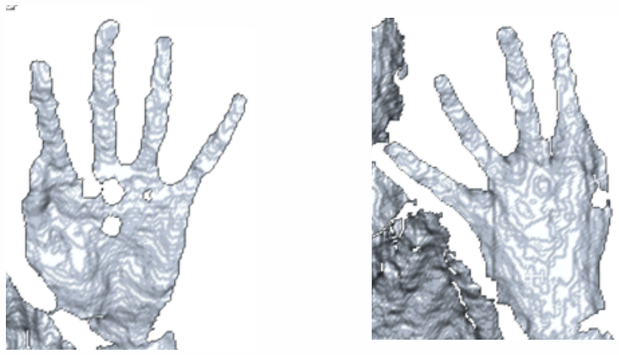
Partial matching

Pairwise Registration

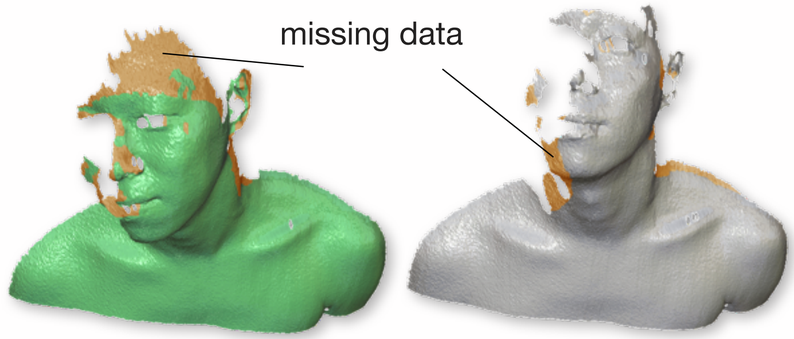
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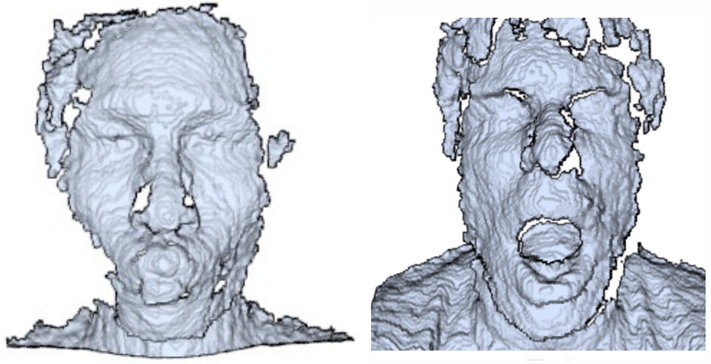
Ambiguity



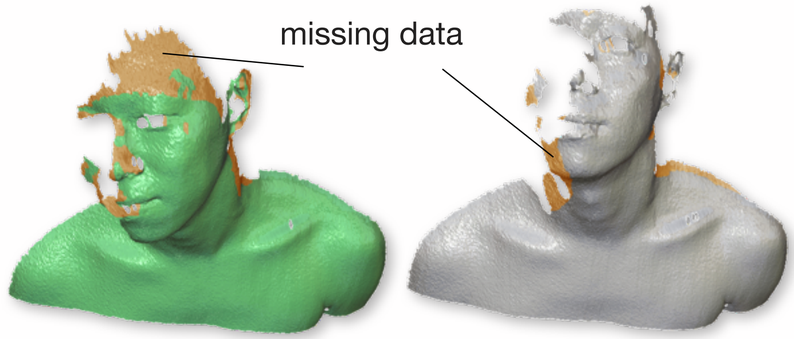
Partial matching

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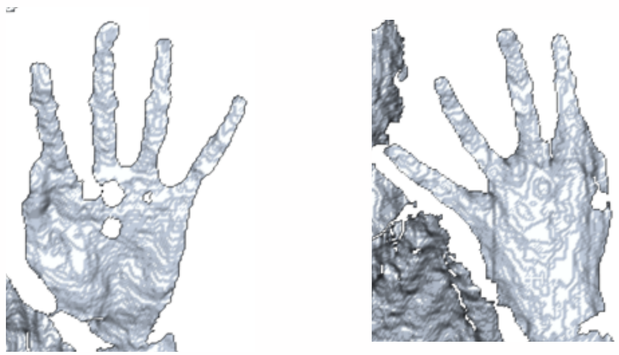
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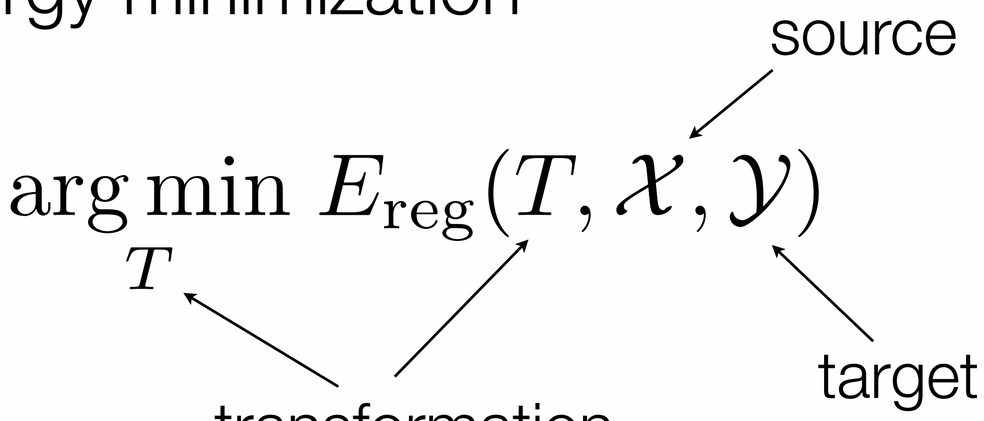
Ambiguity



Illumination changes

Registration

- Registration as energy minimization



Registration

- Registration as energy minimization

$$\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

$$E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y}) = E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) + E_{\text{prior}}(T)$$

Alignment Error

How do we measure the quality of the alignment?

Registration

- Registration as energy minimization

$$\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

$$E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y}) = E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) + E_{\text{prior}}(T)$$

Alignment Error

How do we measure the quality of the alignment?

Transformation Error

What transformations are allowed / good?

Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y})$$

Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function

Registration

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distance function



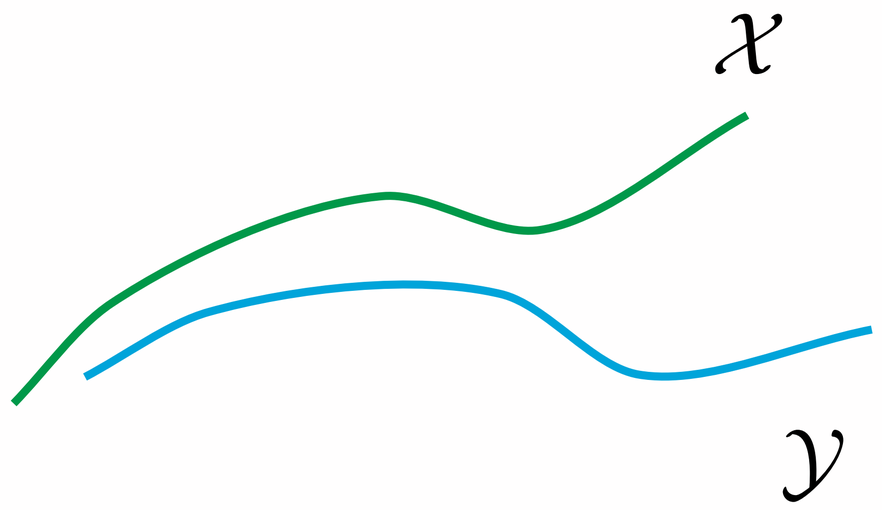
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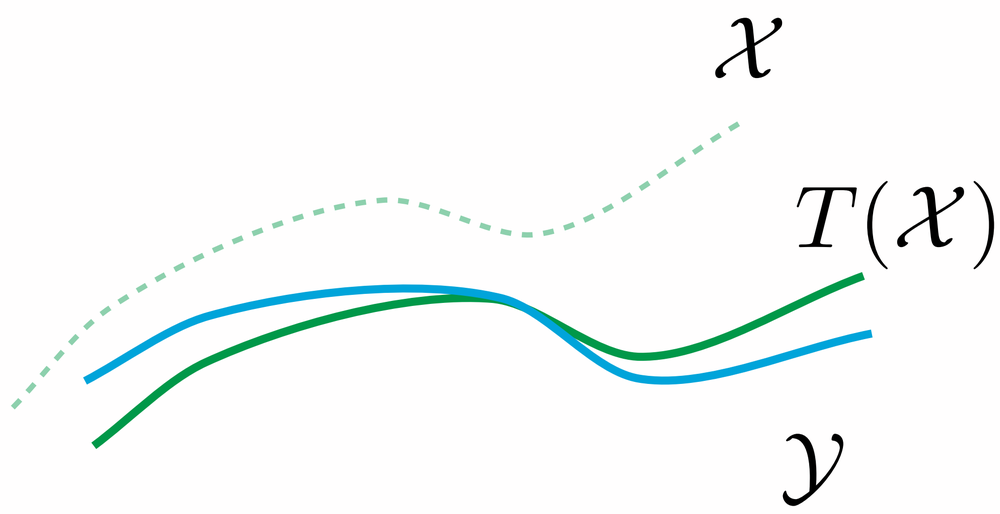
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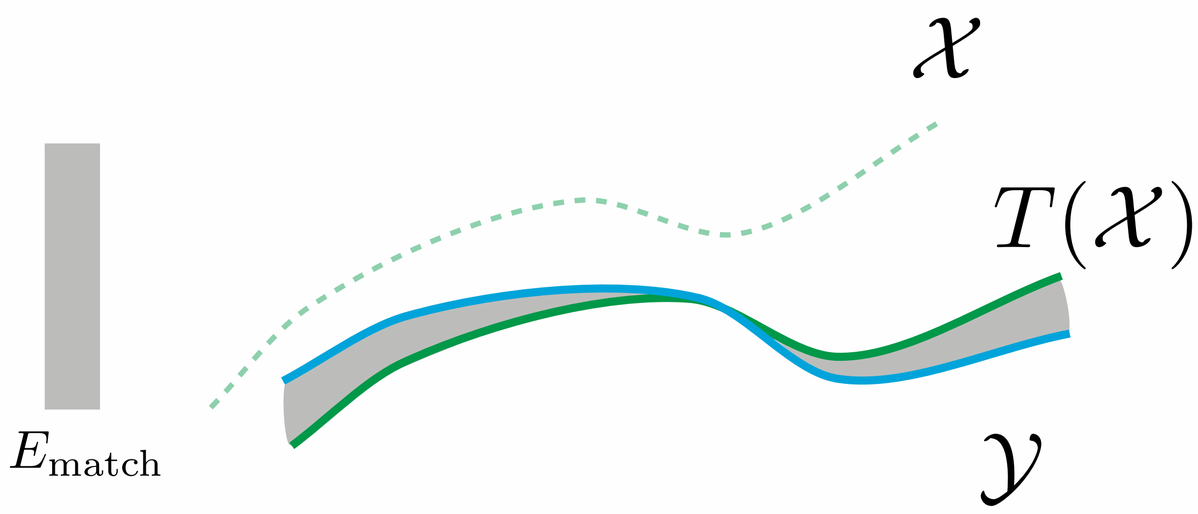
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- Alignment Error

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$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



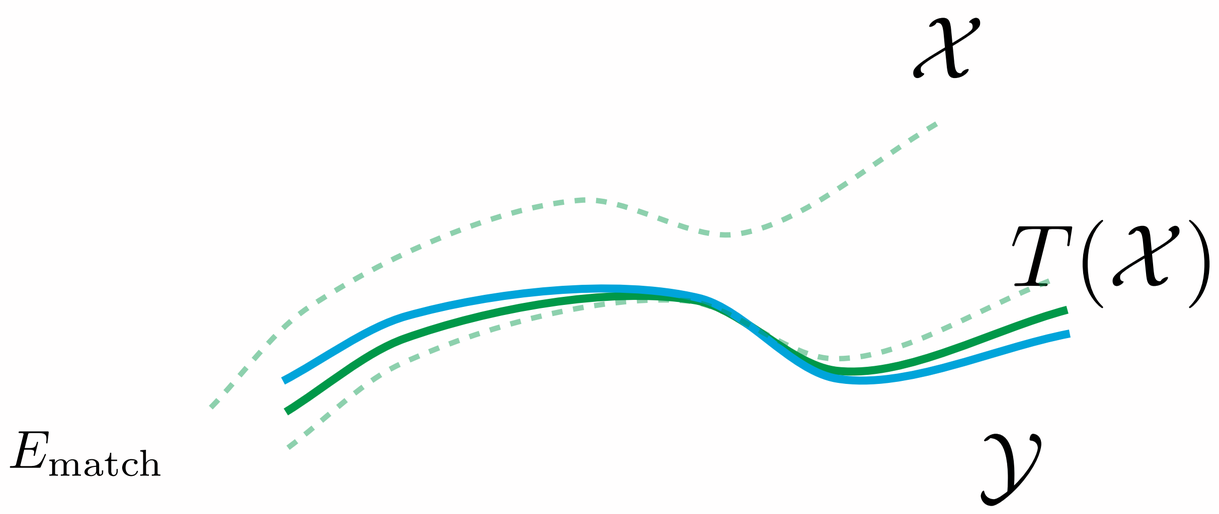
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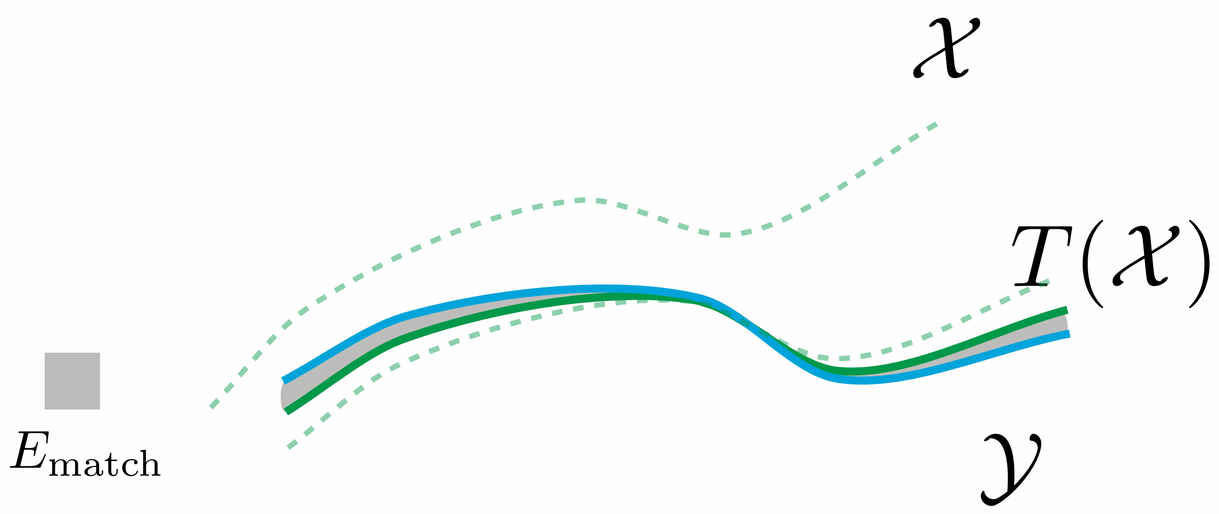
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Registration

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Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



elastic

Registration

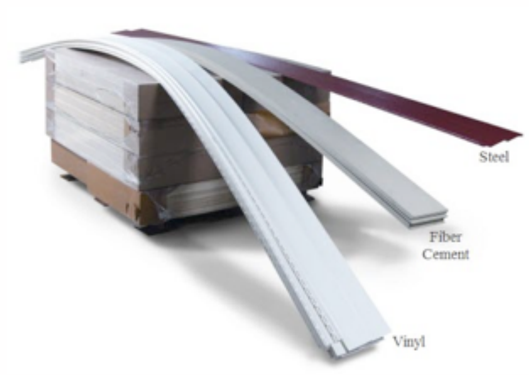
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



articulated



elastic

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



elastic



articulated



composite

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



articulated



composite



elastic



fluid

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elastic



fluid

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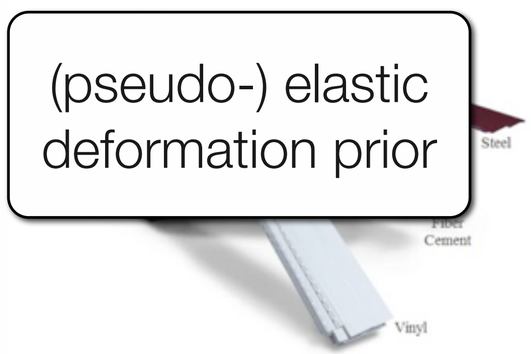
rigid



articulated



composite



elastic



fluid

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid motion prior

rigid

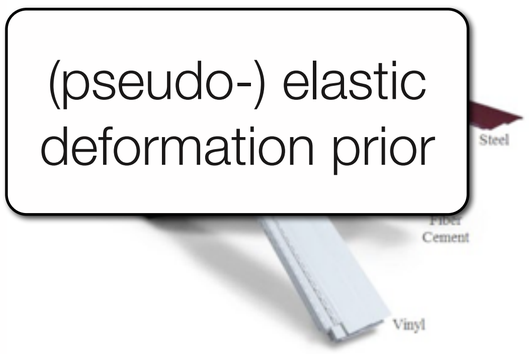


skeletal model prior

articulated



composite



(pseudo-) elastic deformation prior

elastic



fluid

Registration

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rigid motion prior

rigid



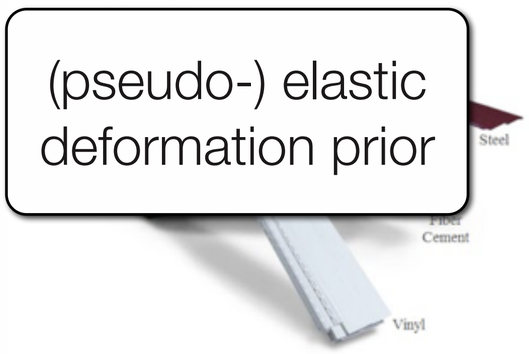
skeletal model prior

articulated



data-driven prior

composite



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elastic



fluid

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rigid motion prior

rigid



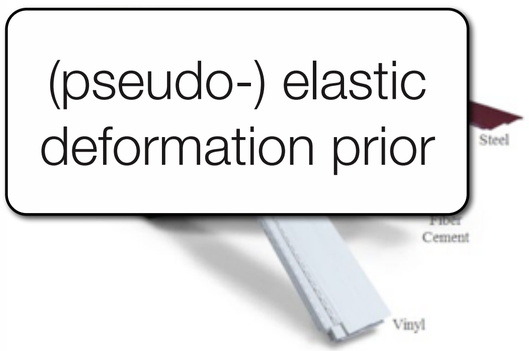
skeletal model prior

articulated



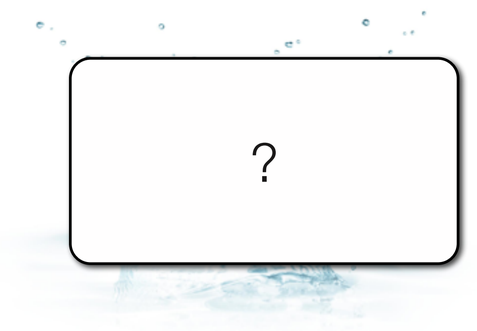
data-driven prior

composite



(pseudo-) elastic deformation prior

elastic



?

fluid

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rigid motion prior

rigid



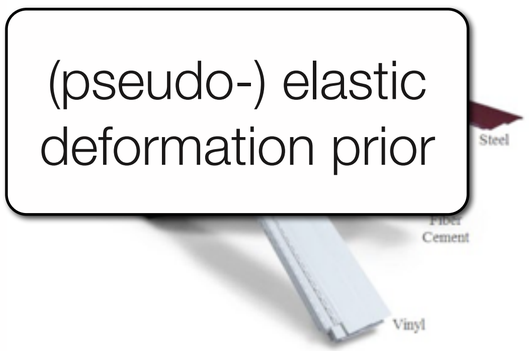
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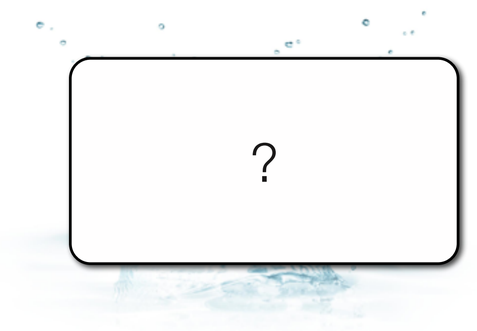
data-driven prior

composite



(pseudo-) elastic deformation prior

elastic



?

fluid

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization

$$\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

source

transformation

target

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  - typically non-linear

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$(\mathcal{X}, \mathcal{Y})$

E_{reg}

$\mathcal{Z}_1 = T_1(\mathcal{X})$

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Input

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Registration

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$\dots$

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Output

$T = T_n$

Overview

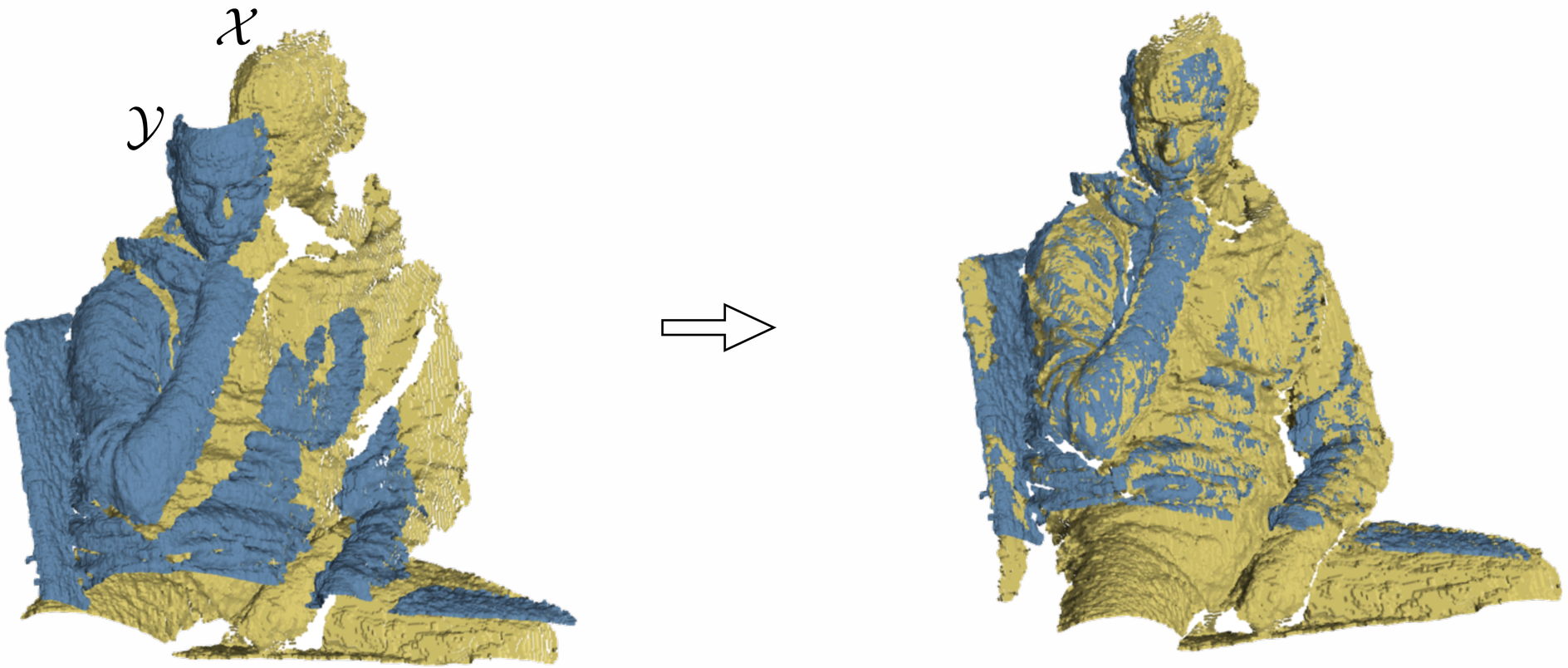
Introduction

- Registration
  - 
  - 3D Geometry
  - 2D Images
  - Combined 2D/3D
  - Robust Registration
  - Q&A
- Applications
  - Rigid Scanning
  - Non-rigid Modeling
  - Realtime Face Tracking

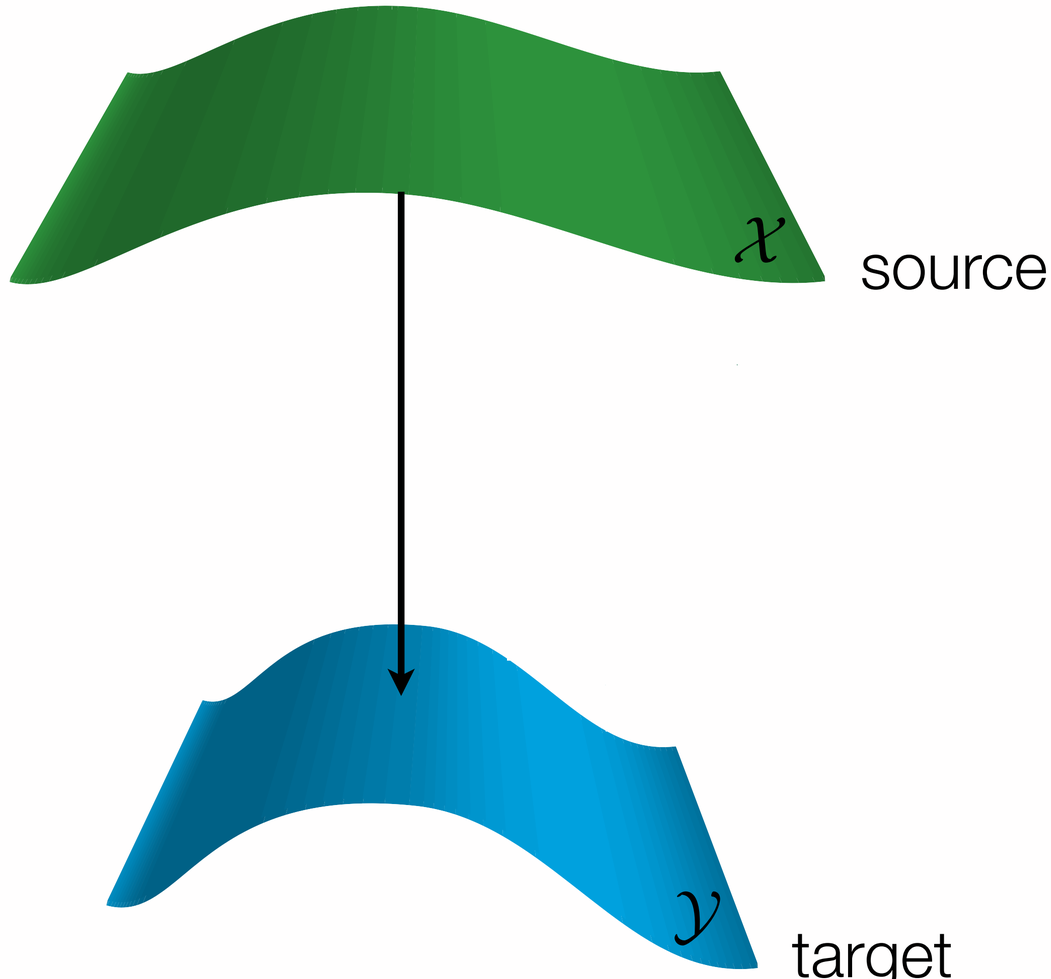
Conclusions

Q&A

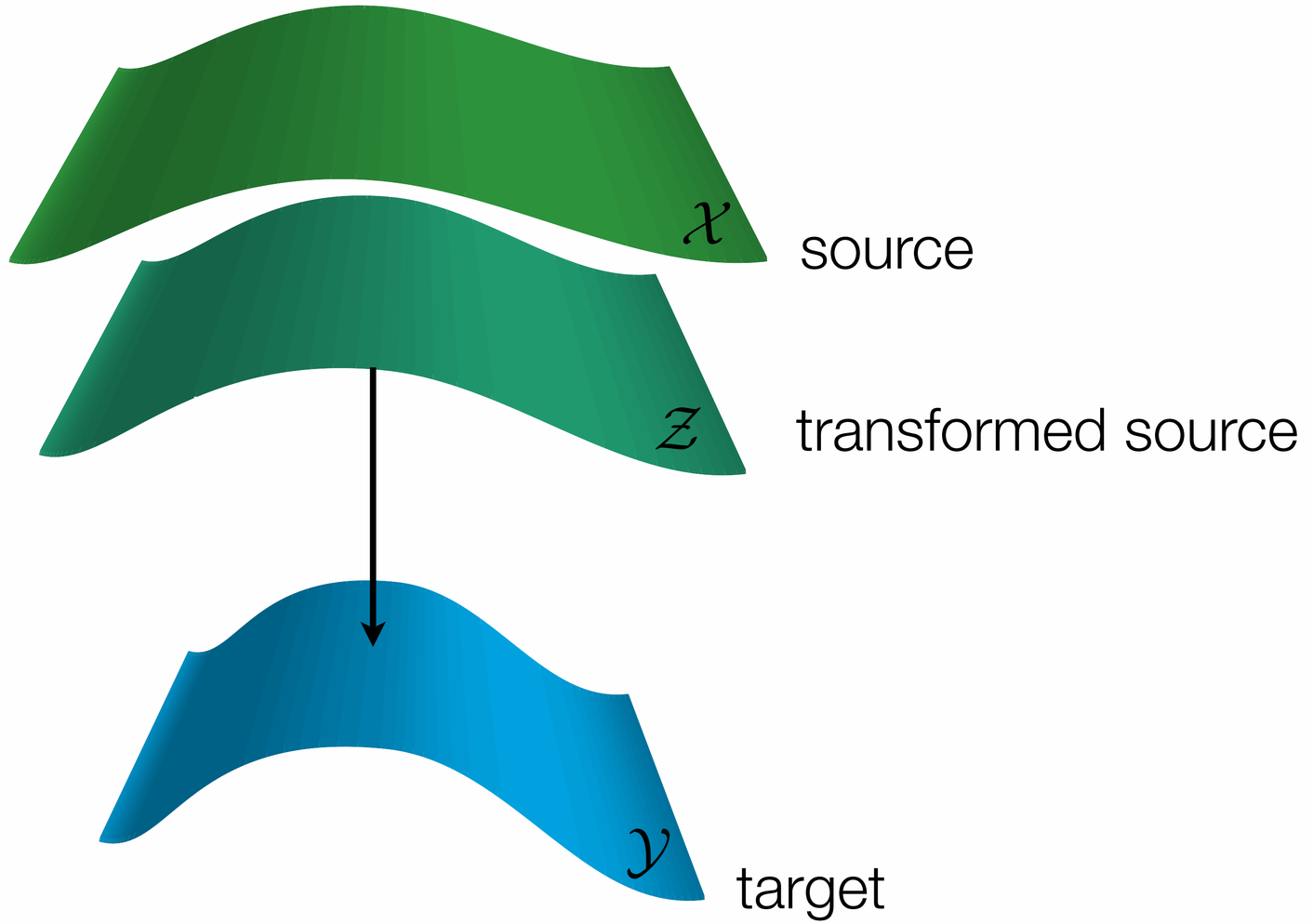
3D Registration



3D Registration



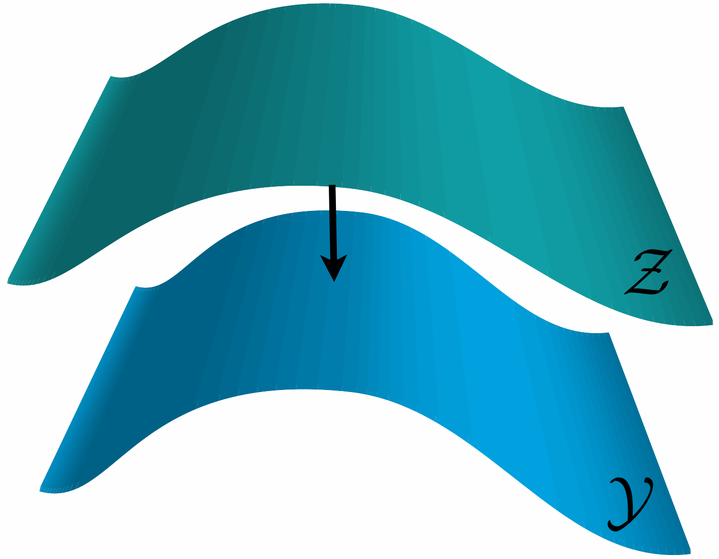
3D Registration



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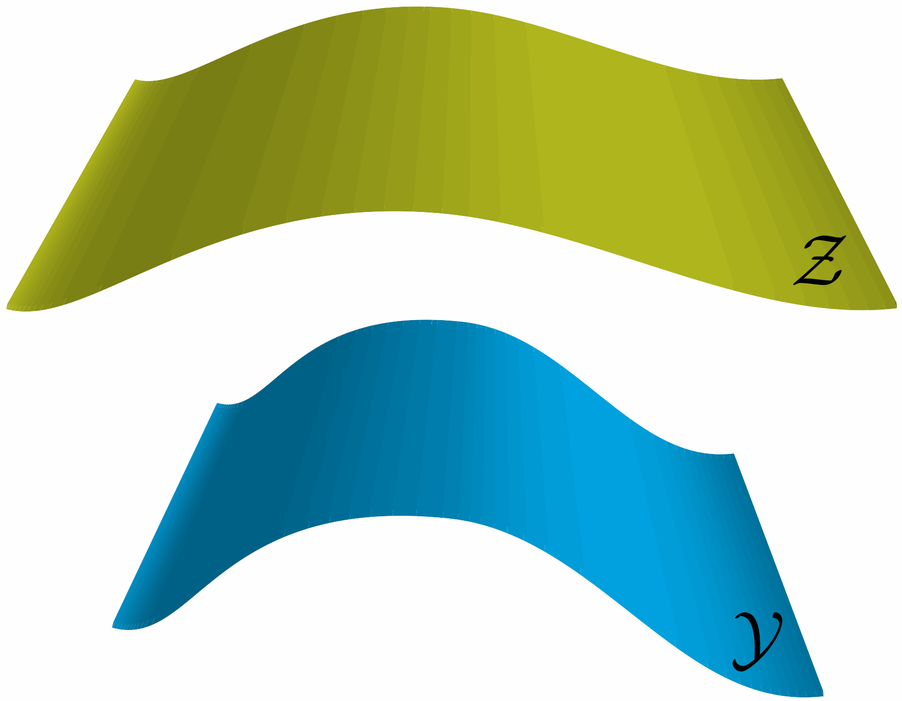
source



transformed source

target

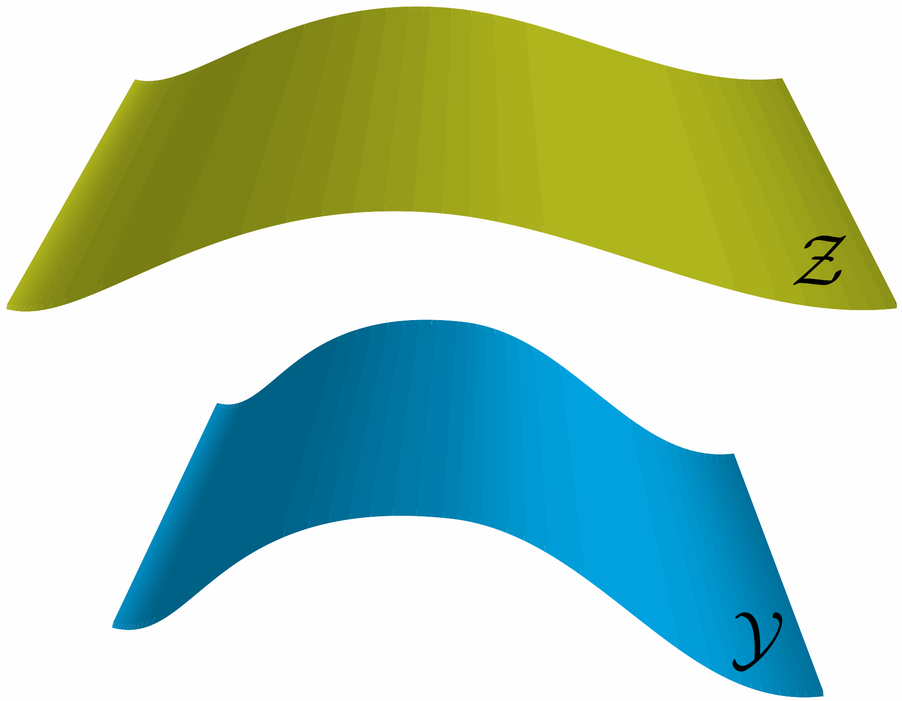
3D Registration - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z}$$

3D Registration - Matching

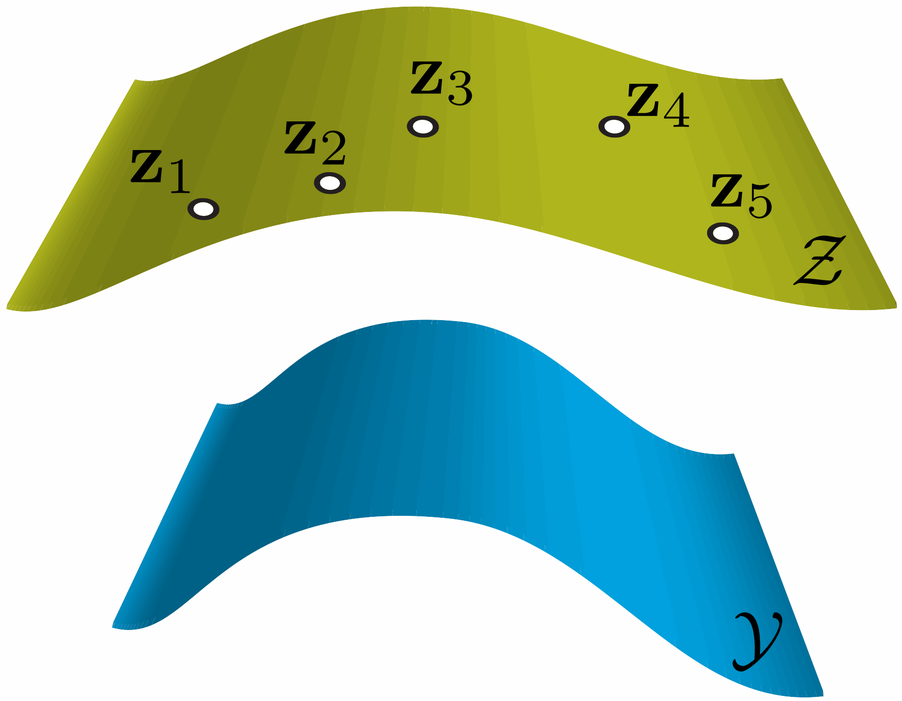


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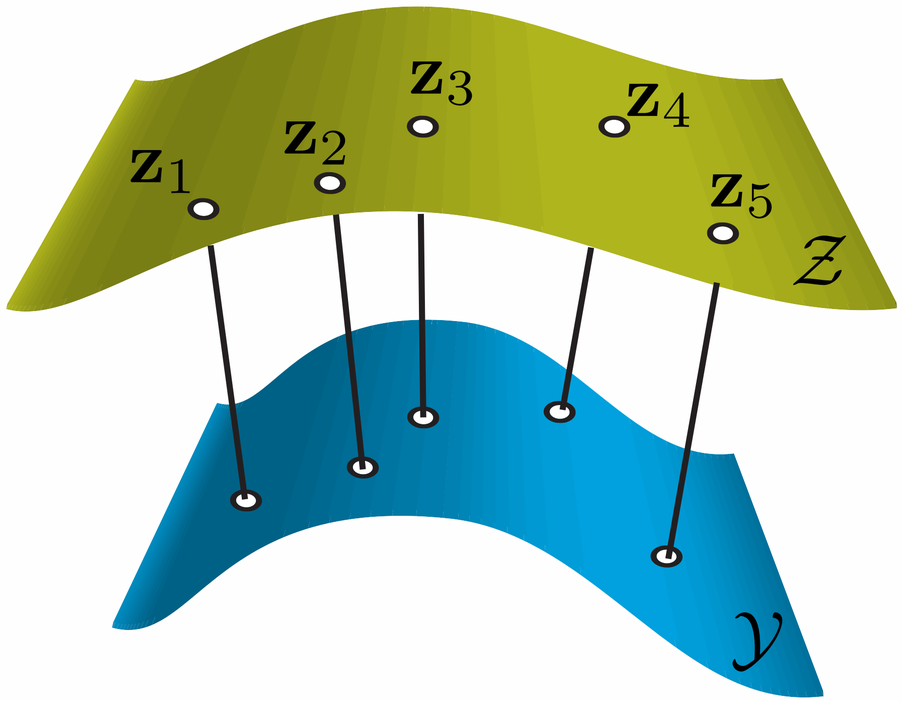
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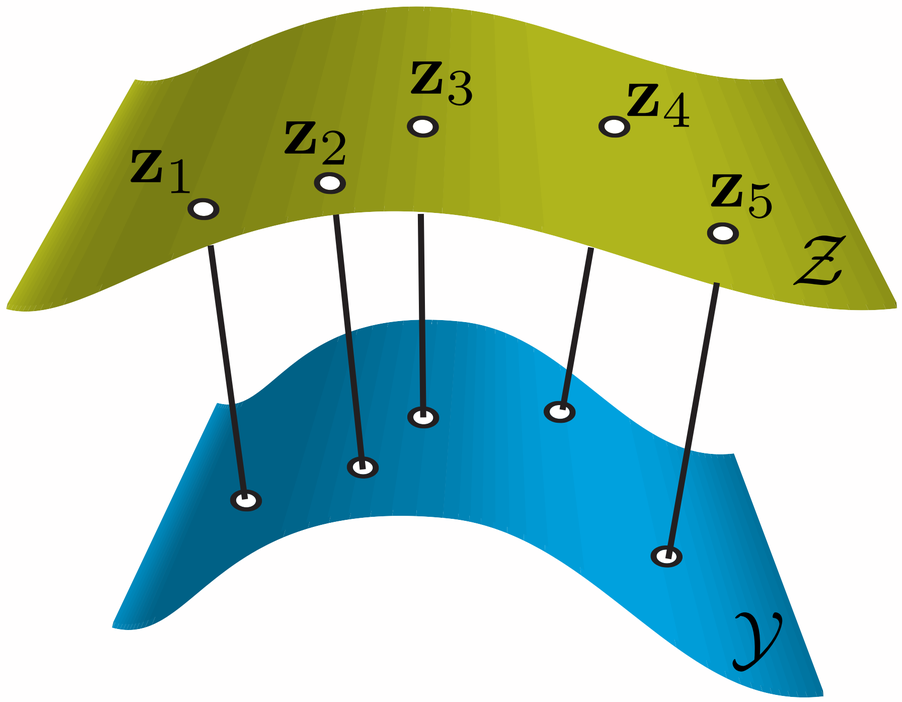
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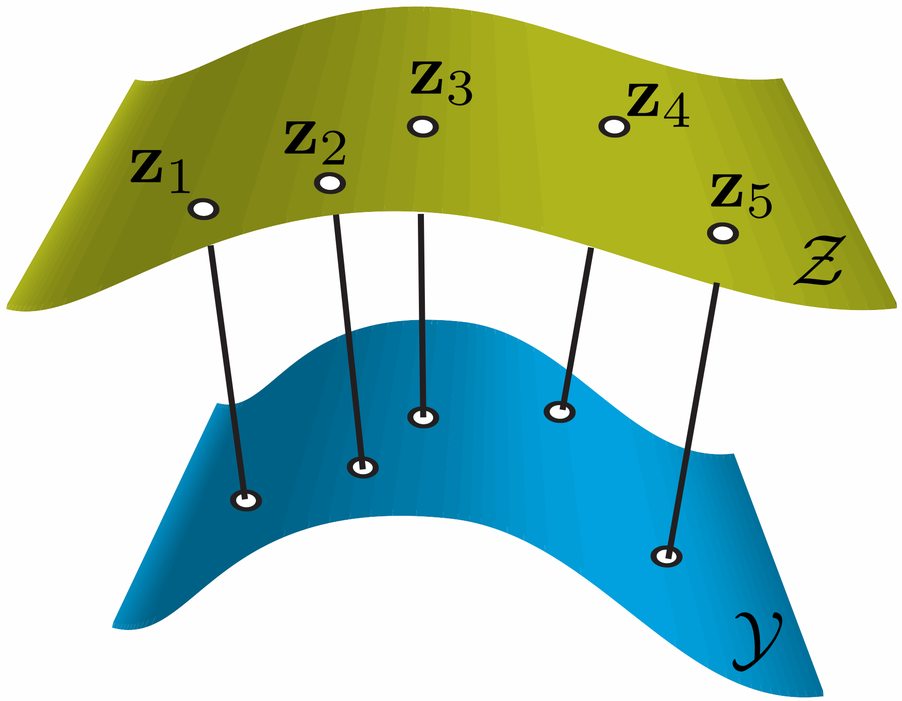


discretized matching cost based on point correspondences

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3D Registration - Matching



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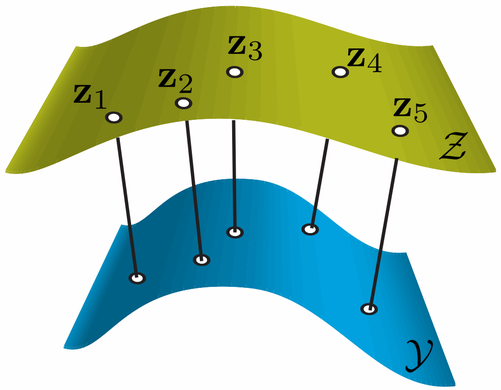
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$$\Downarrow$$

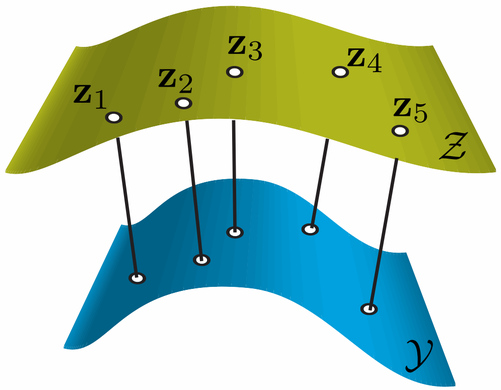
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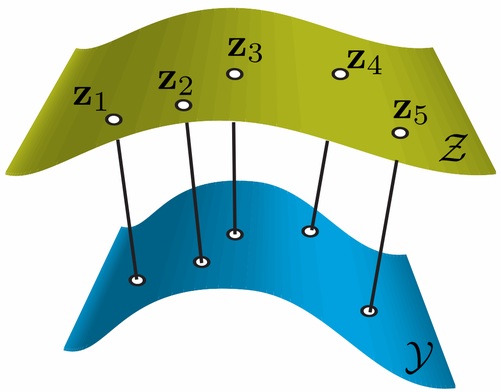
3D Registration - Matching



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transformed point on source

3D Registration - Matching

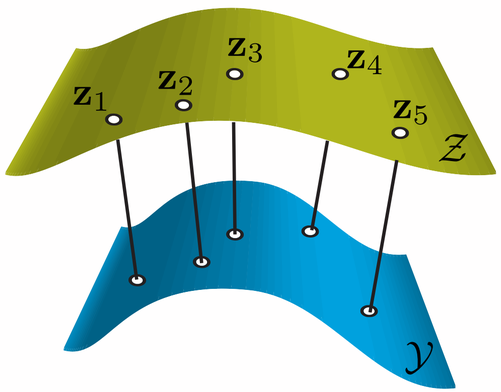


corresponding point on target

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

transformed point on source

3D Registration - Matching



correspondence weight

corresponding point on target

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transformed point on source

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

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  - **Step 1:** find correspondences using closest points for fixed transformation

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Iterate until convergence

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DEMO

3D Registration - Example



- Iterative Closest Point (ICP) Algorithm
  - (approximate) closest points→ (more) **efficient** data structures

3D Registration - Example

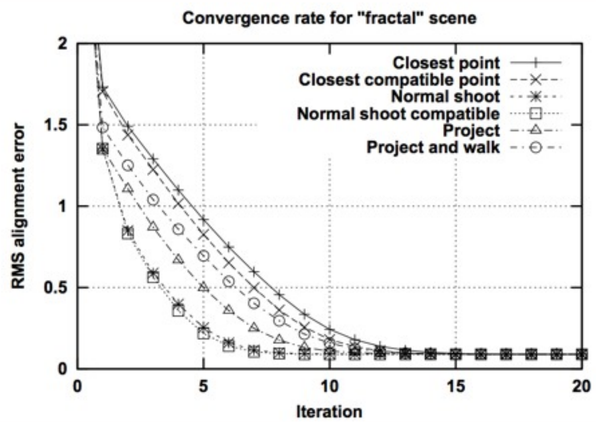
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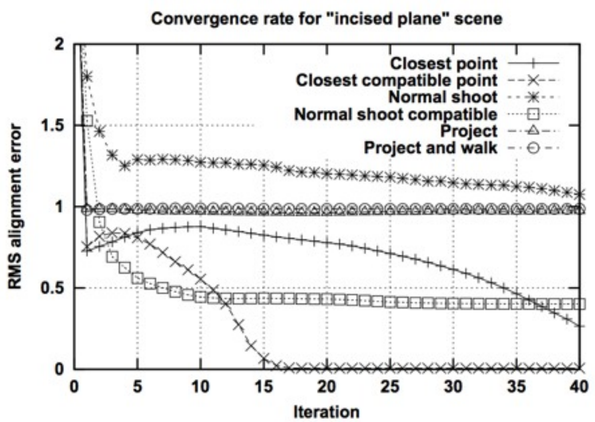
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Rusinkiewicz, Levoy: **Efficient Variants of the ICP Algorithm**, *3D Digital Imaging and Modeling, 2001*

3D Registration - Example

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- Side Remark: Error norm
  - squared Euclidean distance is sensitive to outliers
  - robust norms reduce this sensitivity

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



articulated



composite

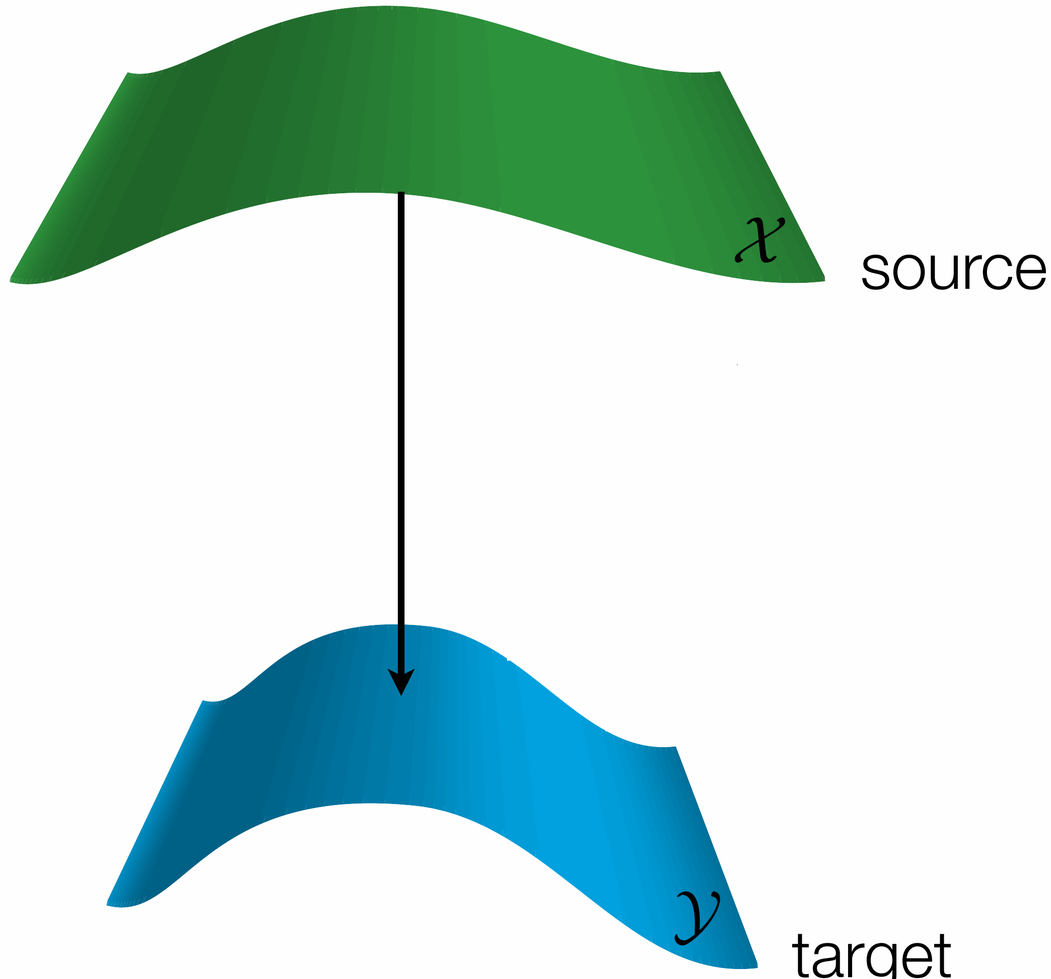


elastic

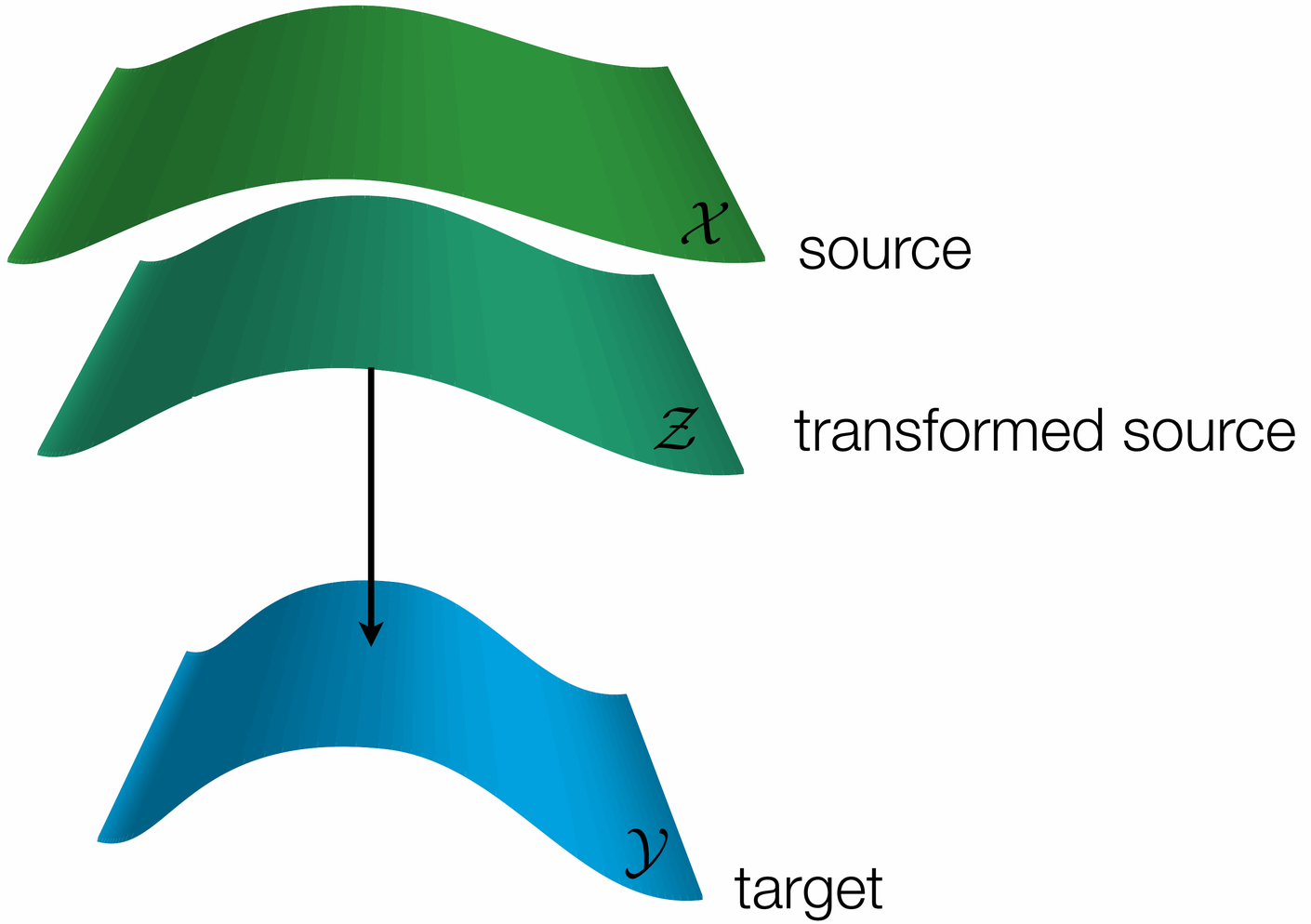


fluid

3D Registration - Prior



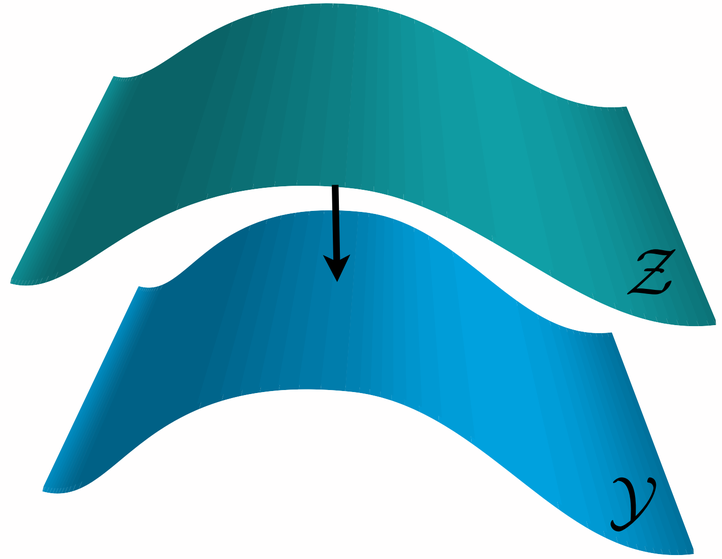
3D Registration - Prior



3D Registration - Prior



source



transformed source

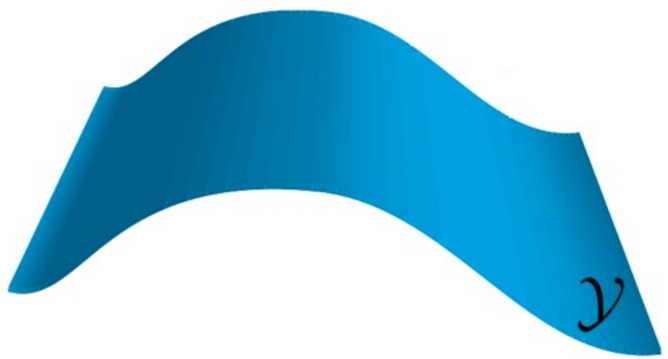
target

3D Registration - Prior

- Why priors?

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

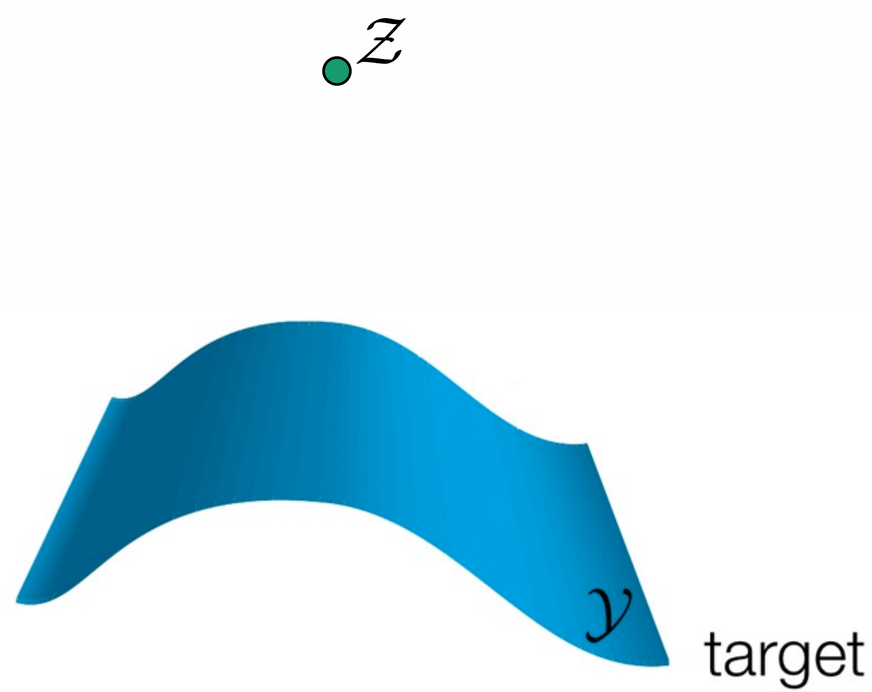




3D Registration - Prior

- Why priors?

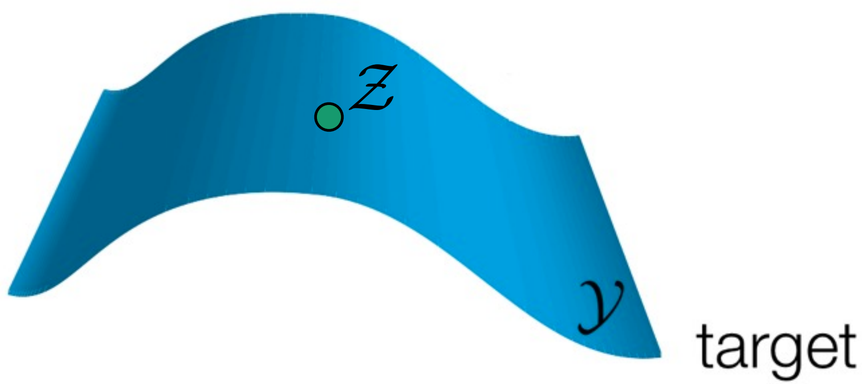
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3D Registration - Prior

- Why priors?

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

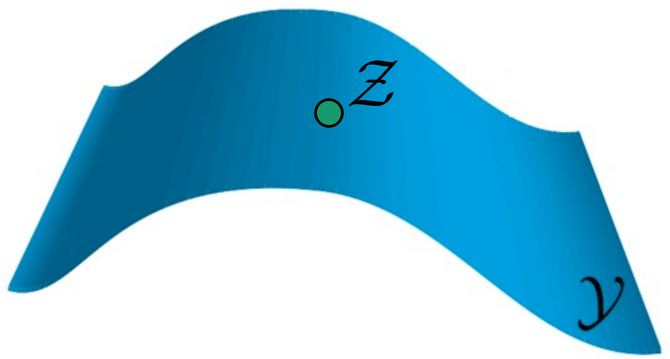


3D Registration - Prior

- Why priors?

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z} = 0!!!$$



3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid

articulated



composite



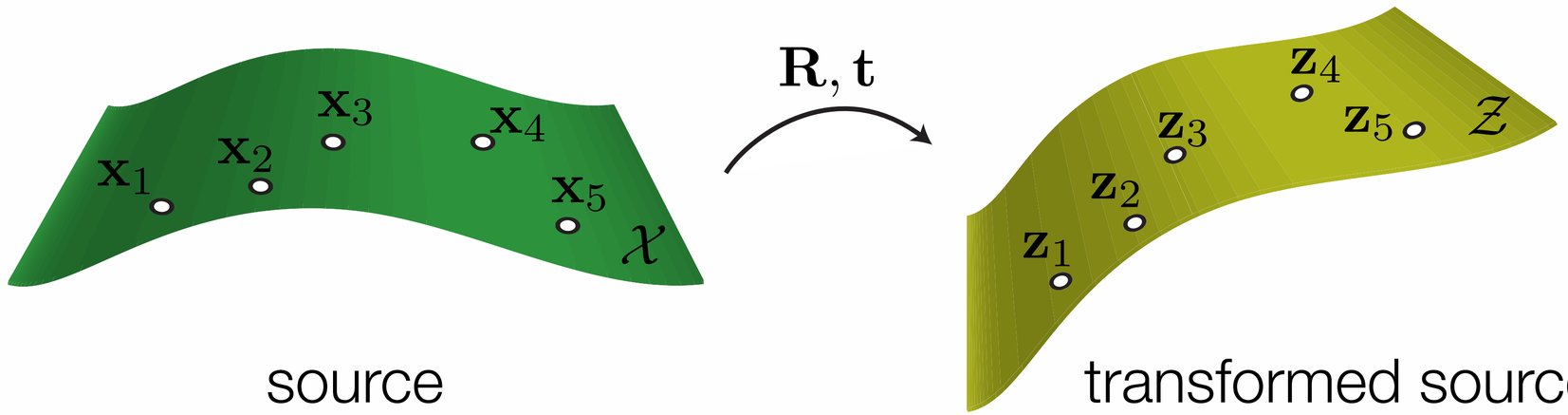
elastic



fluid

3D Registration - Prior

- Global Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

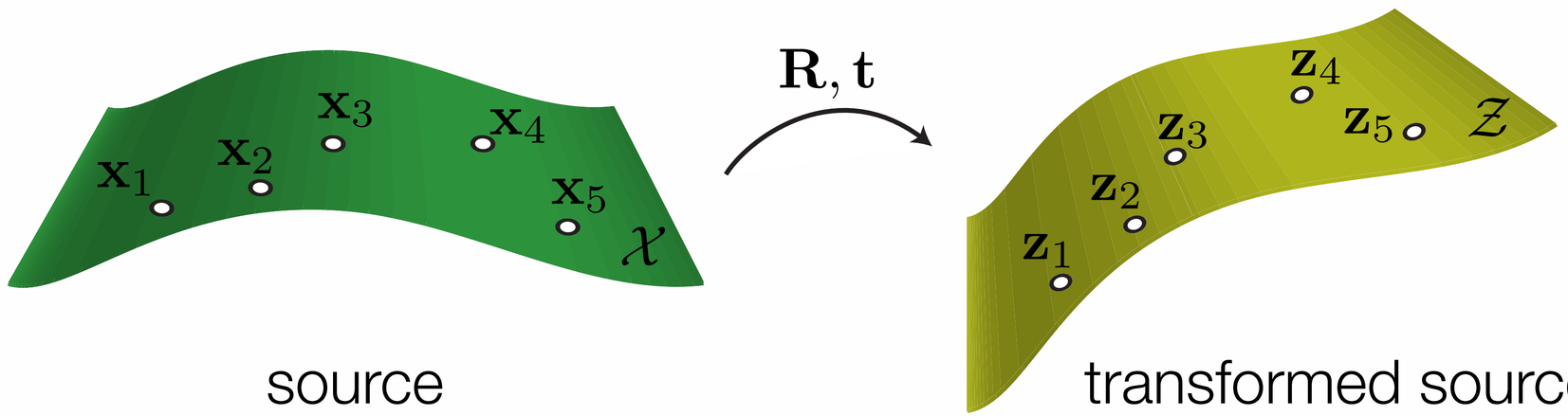


source

transformed source

3D Registration - Prior

- Global Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



source transformed source

$$E_{\text{prior}}(Z, \mathbf{R}, \mathbf{t}) = \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid

articulated



composite



elastic



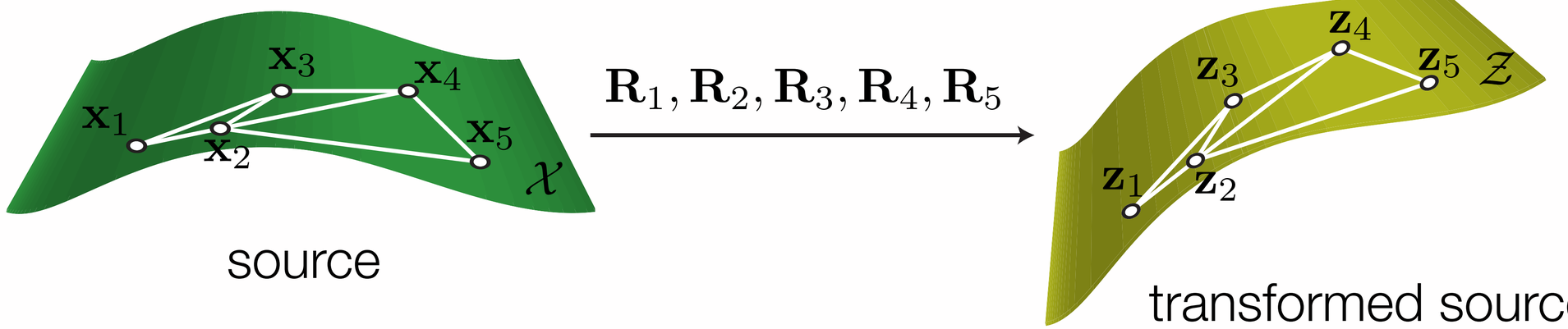
fluid

3D Registration - Prior

- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

3D Registration - Prior

- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

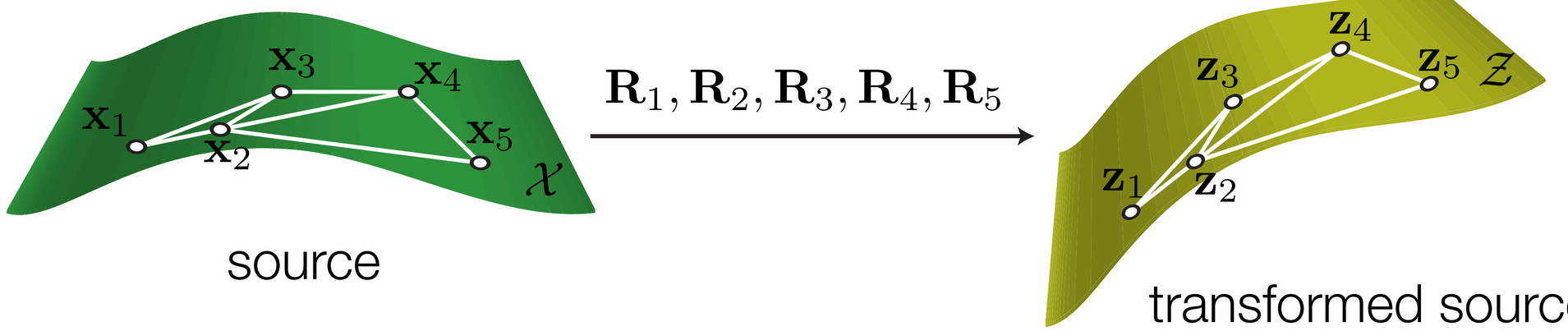


source

transformed source

3D Registration - Prior

- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



source

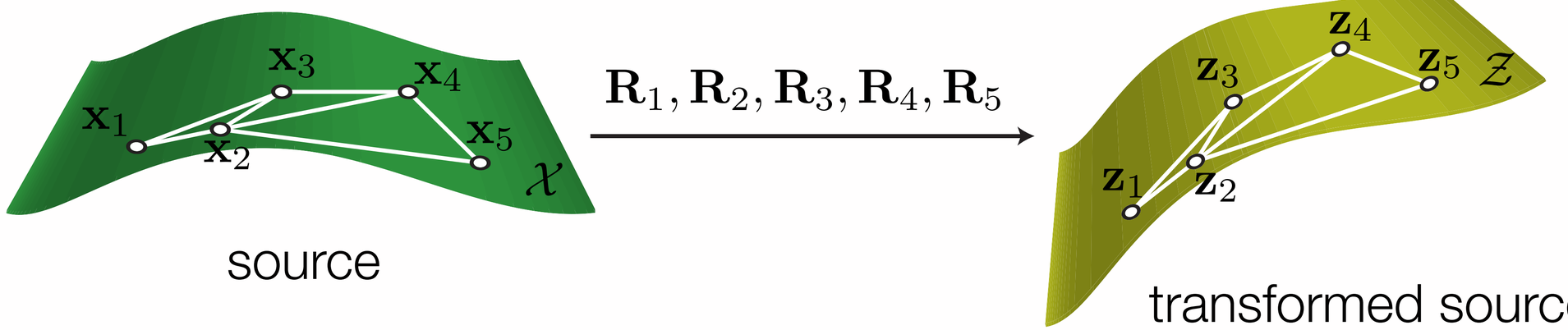
transformed source

$$E_{\text{prior}}(Z, \{\mathbf{R}_i\}) = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

Sorkine, Alexa: **As-Rigid-As-Possible Surface Modeling**, *SGP 2007*

3D Registration - Prior

- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



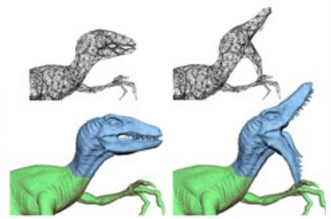
source

transformed source

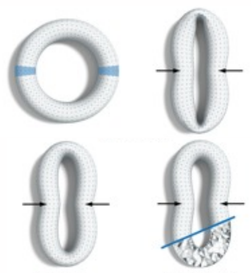
DEMO

3D Registration - Prior

- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



Sumner, Schmid, Pauly:
Embedded Deformation for Shape Manipulation
ACM SIGGRAPH 2007



Bouaziz, Deuss, Schwartzburg, Weise, Pauly:
Shape-Up: Shaping Discrete Geometry With Projection
SGP 2012

primo

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



articulated



composite



elastic

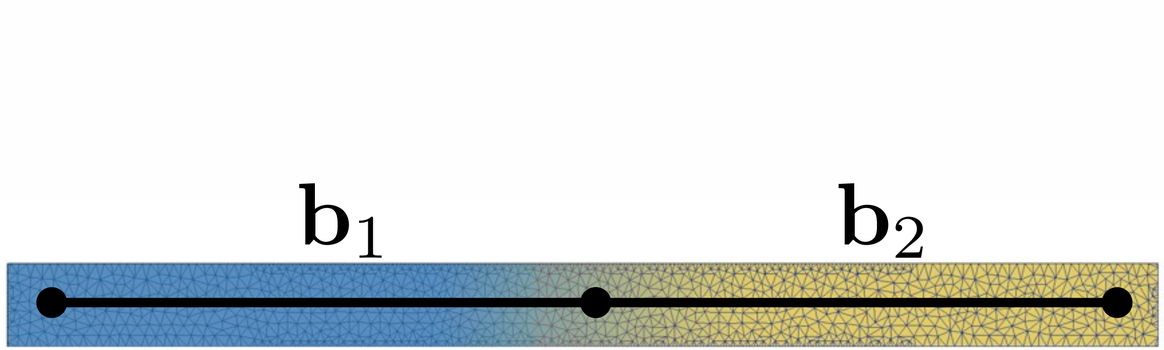


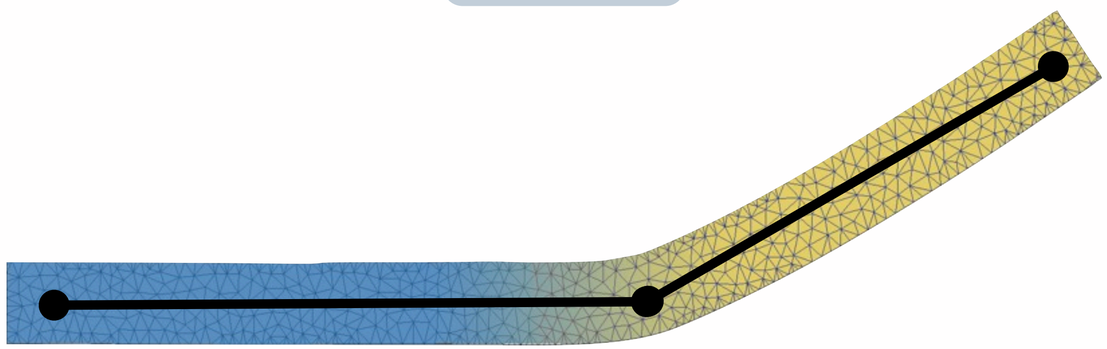
fluid

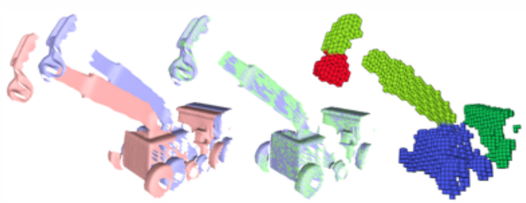
3D Registration - Prior

- Linear Blend Skinning

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$





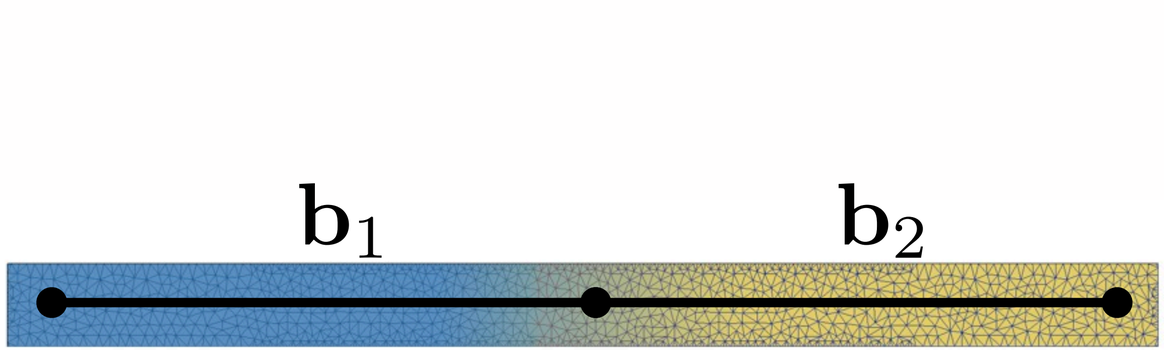


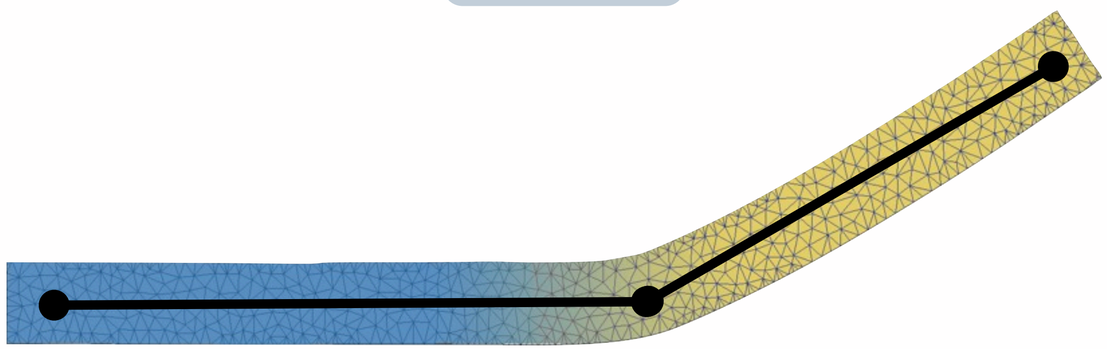
Chang, Zwicker: **Range Scan Registration Using Reduced Deformable Models**
EUROGRAPHICS 2009

3D Registration - Prior

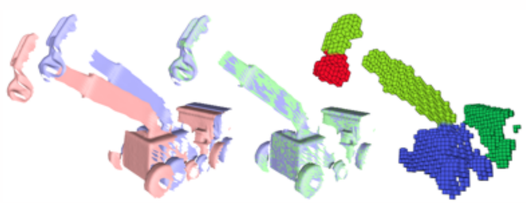
- Linear Blend Skinning

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$





$$E_{\text{prior}}(Z, \mathbf{R}_j, \mathbf{t}_j) = \sum_{i=1}^{n} \sum_{j \in \mathcal{B}} w_{i,j} \left\| \mathbf{z}_i - (\mathbf{R}_j \mathbf{x}_i + \mathbf{t}_j)) \right\|_2^2$$

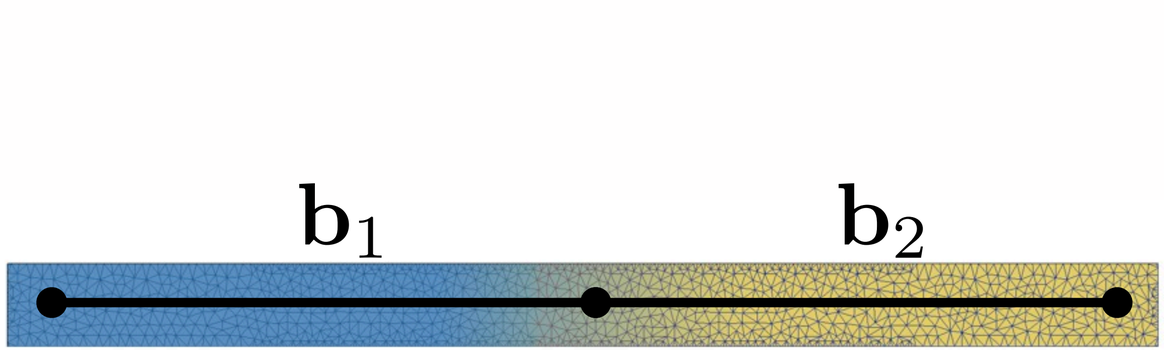


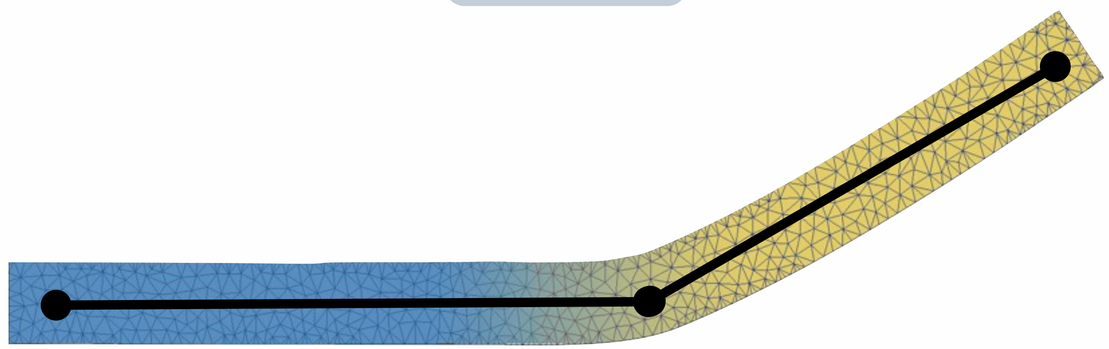
Chang, Zwicker: **Range Scan Registration Using Reduced Deformable Models**
EUROGRAPHICS 2009

3D Registration - Prior

- Linear Blend Skinning

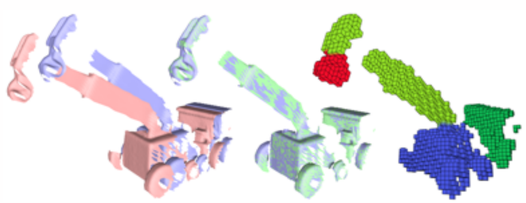
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rigid transformations

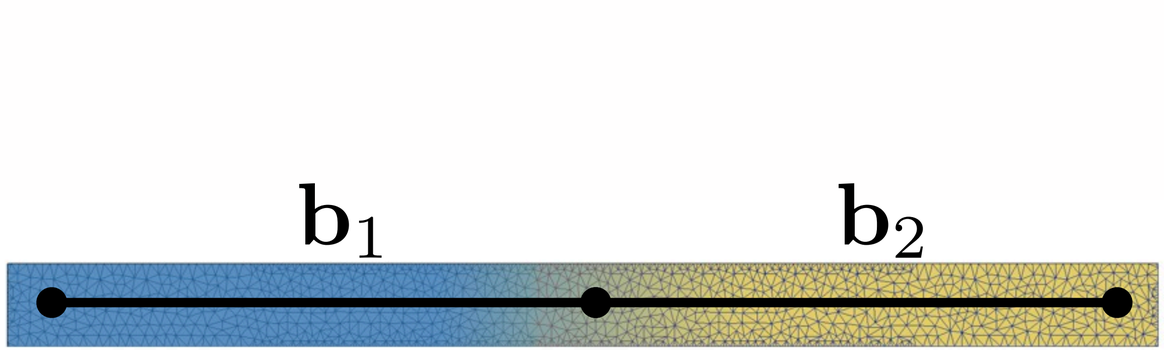


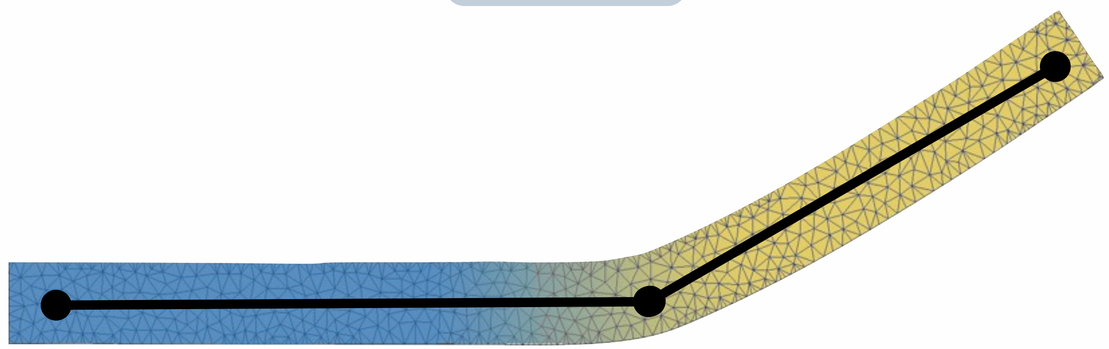
Chang, Zwicker: **Range Scan Registration Using Reduced Deformable Models**
EUROGRAPHICS 2009

3D Registration - Prior

- Linear Blend Skinning

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

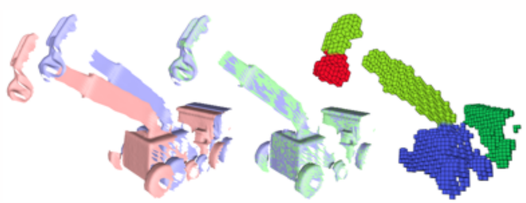




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blended

rigid transformations

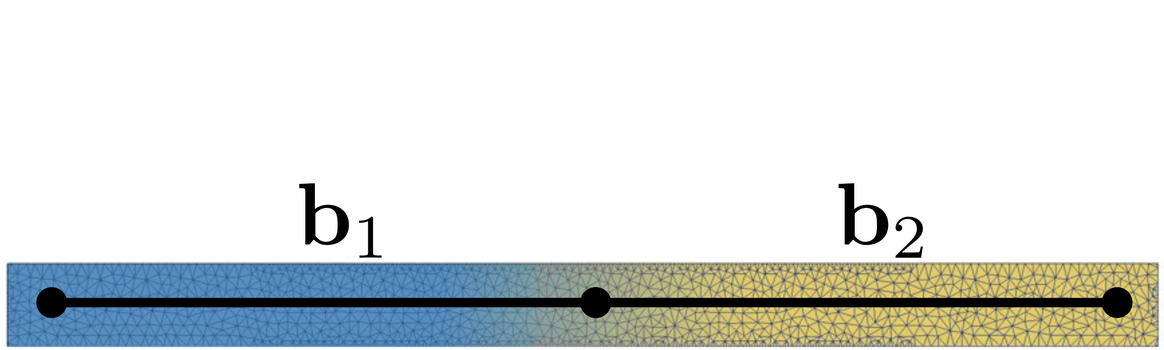


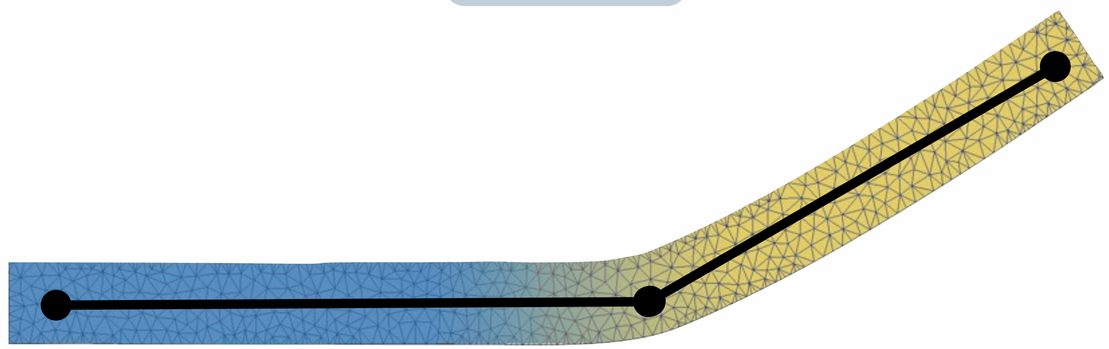
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3D Registration - Prior

- Linear Blend Skinning

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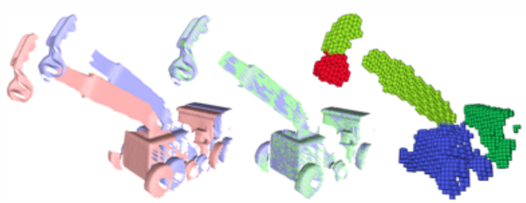




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blended

rigid transformations



Chang, Zwicker: **Range Scan Registration Using Reduced Deformable Models**
EUROGRAPHICS 2009

DEMO

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid

articulated



composite



elastic



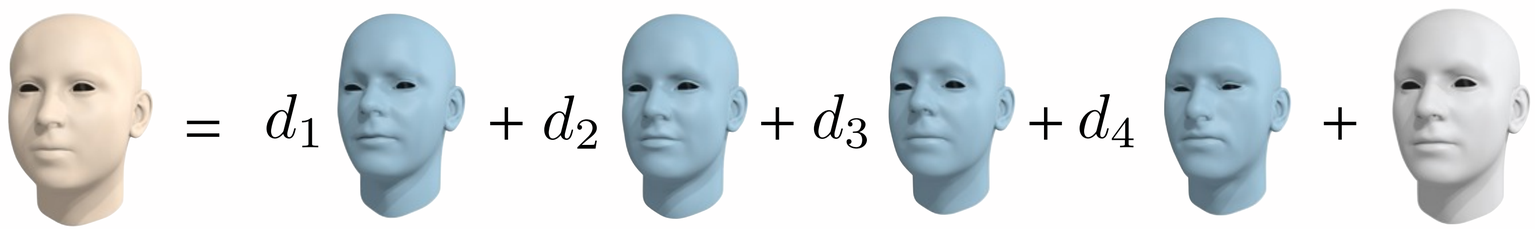
fluid

3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

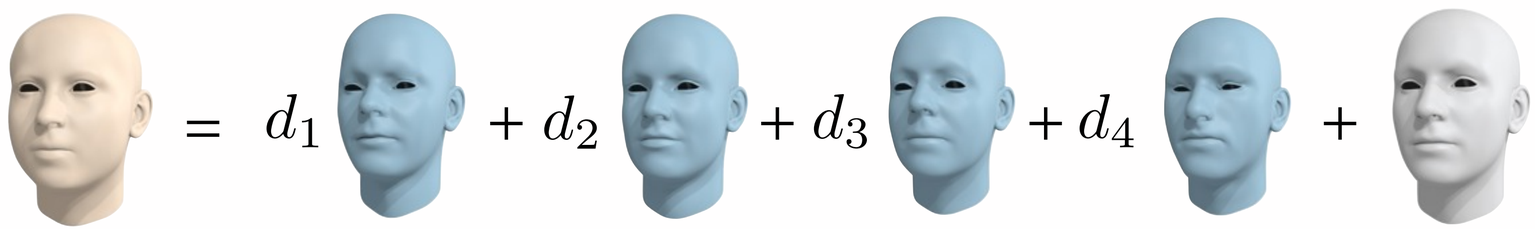
3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



3D Registration - Prior

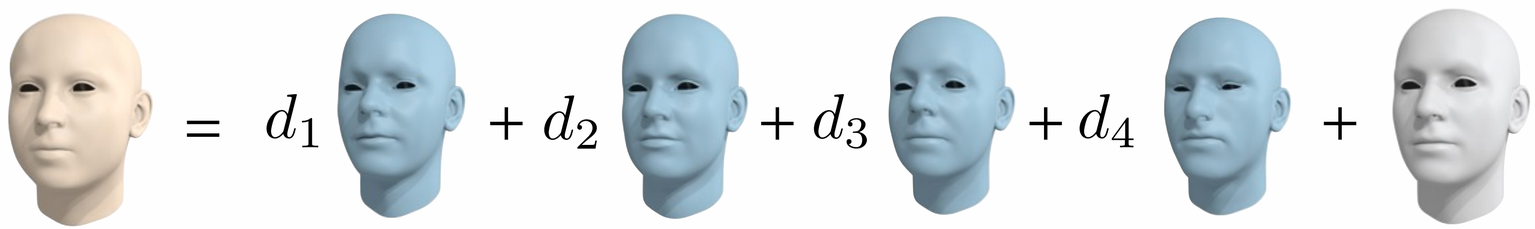
- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



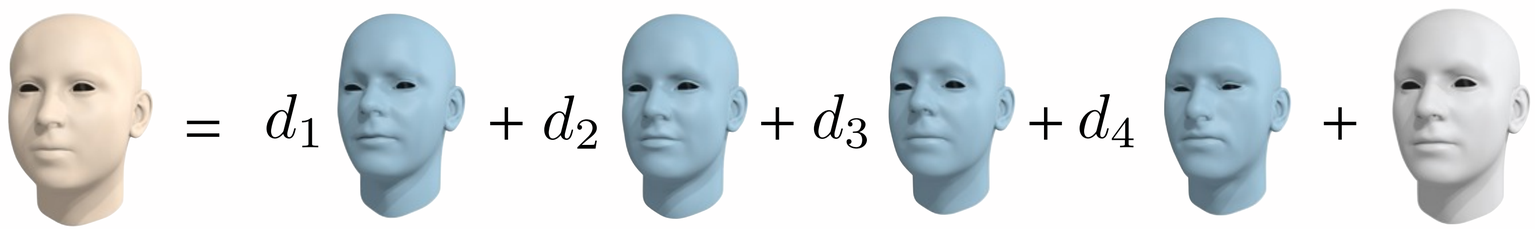
$$= d_1 + d_2 + d_3 + d_4 +$$

$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

mean shape

3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



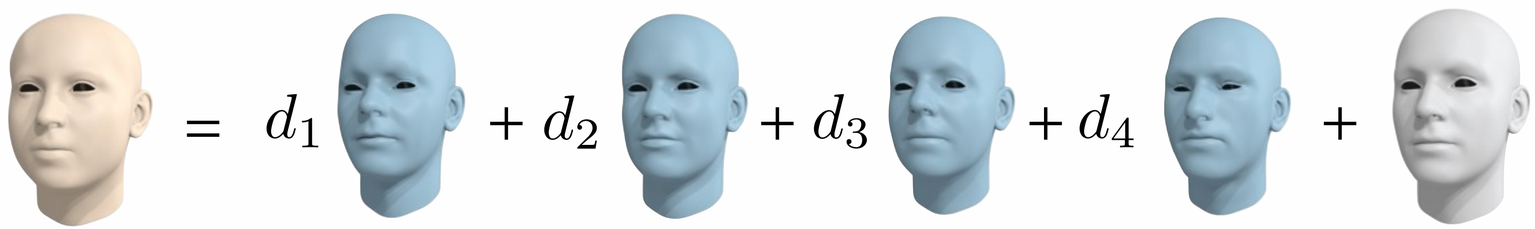
$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

basis

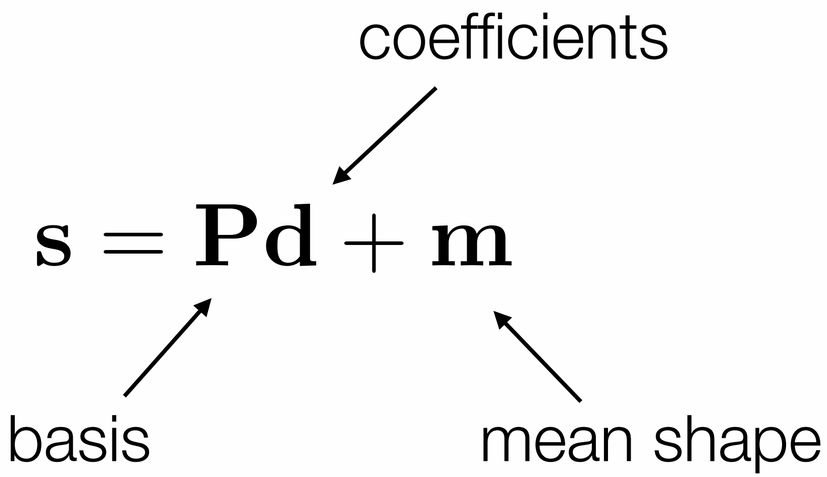
mean shape

3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

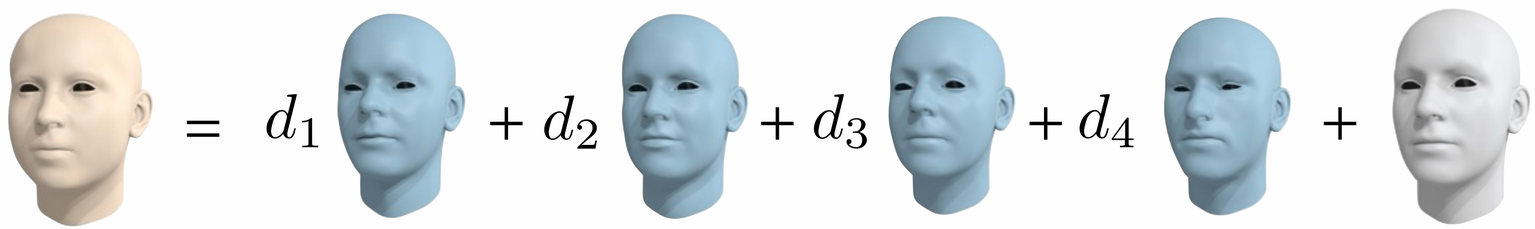


$= d_1 + d_2 + d_3 + d_4 +$



3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

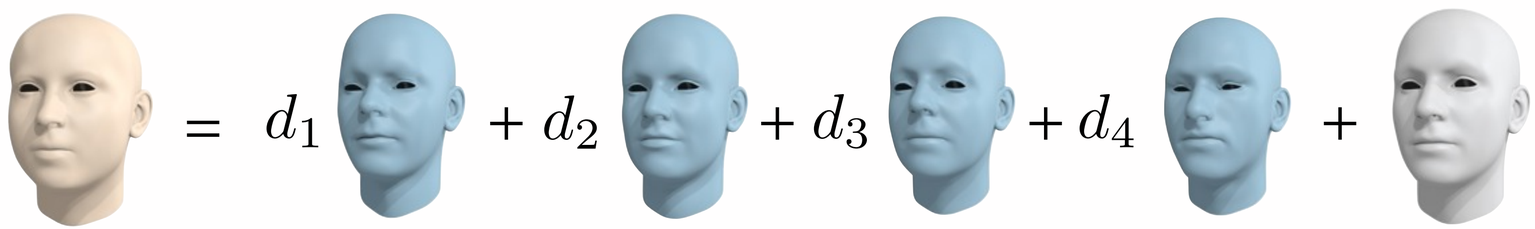


$$= d_1 + d_2 + d_3 + d_4 +$$

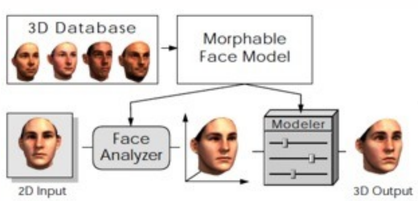
$$E_{\text{prior}}(Z, \mathbf{d}) = \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{P}(i,:)\mathbf{d} + \mathbf{m}(i))\|_2^2$$

3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

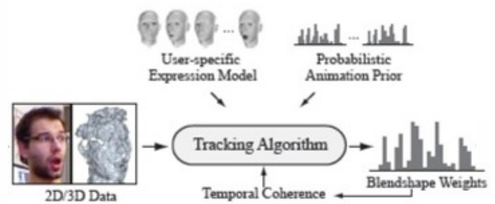


$$E_{\text{prior}}(Z, \mathbf{d}) = \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{P}(i,:)\mathbf{d} + \mathbf{m}(i))\|_2^2$$



Blanz, Vetter: **A Morphable Model for the Synthesis of 3D Faces**
ACM SIGGRAPH 1999

Weise, Bouaziz, Li, Pauly: **Realtime Performance-based Facial Animation**
ACM SIGGRAPH 2011



3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid

articulated



composite



elastic

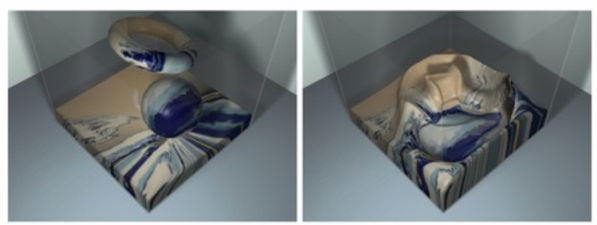


fluid

3D Registration - Prior

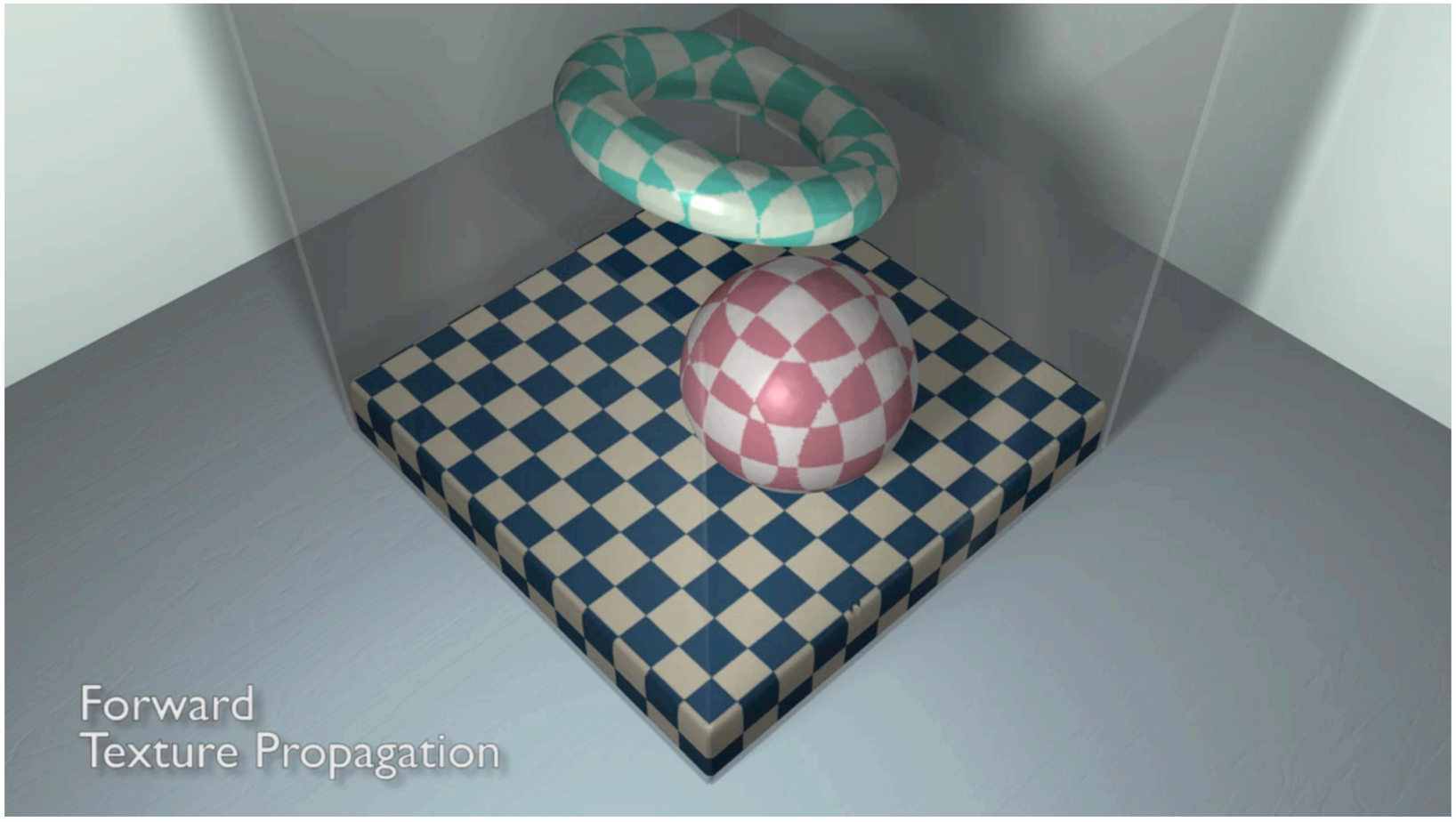
- Evolving topology

Bojsen-Hansen, Li, Wojtan:
Tracking Surfaces with Evolving Topolgy
ACM SIGGRAPH 2012

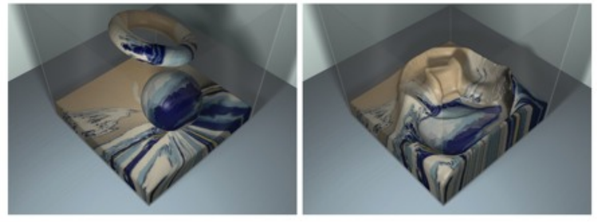


3D Registration - Prior

- Evolving topology



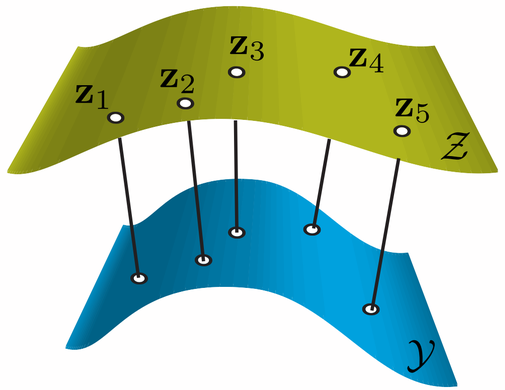
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Tracking Surfaces with Evolving Topolgy
ACM SIGGRAPH 2012



3D Registration - Recap

3D Registration - Recap

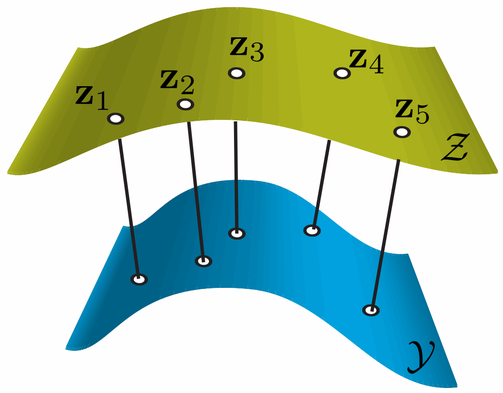
Matching



$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

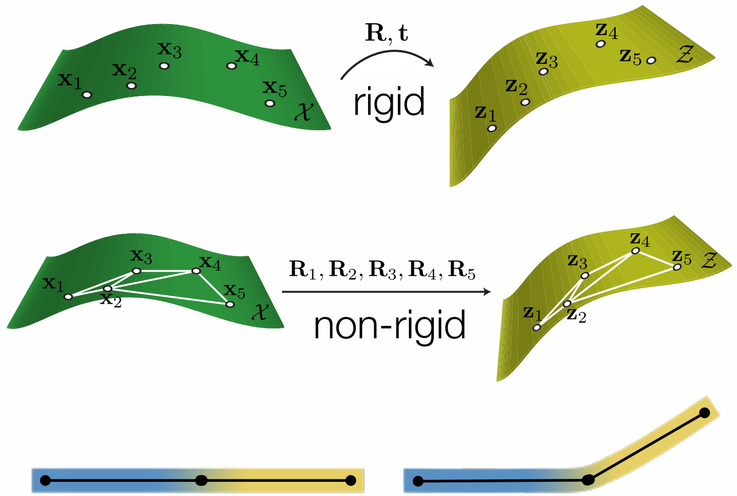
3D Registration - Recap

Matching



$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

Prior



Break (5min)



Overview

Introduction

- Registration
  - 3D Geometry
  - 2D Images
  - Combined 2D/3D
  - Robust Registration
  - Q&A
- Applications
  - Rigid Scanning
  - Non-rigid Modeling
  - Realtime Face Tracking

Conclusions

Q&A

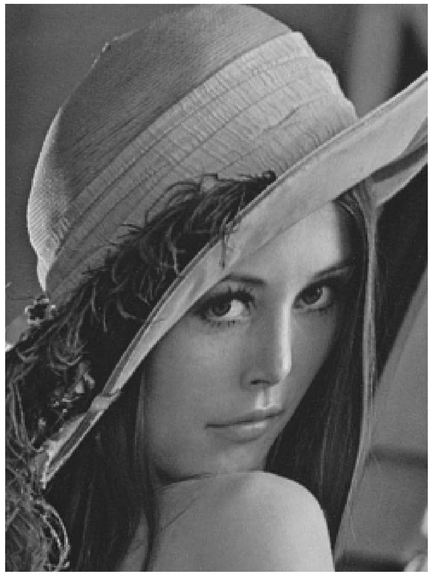
2D Registration

Target

Source



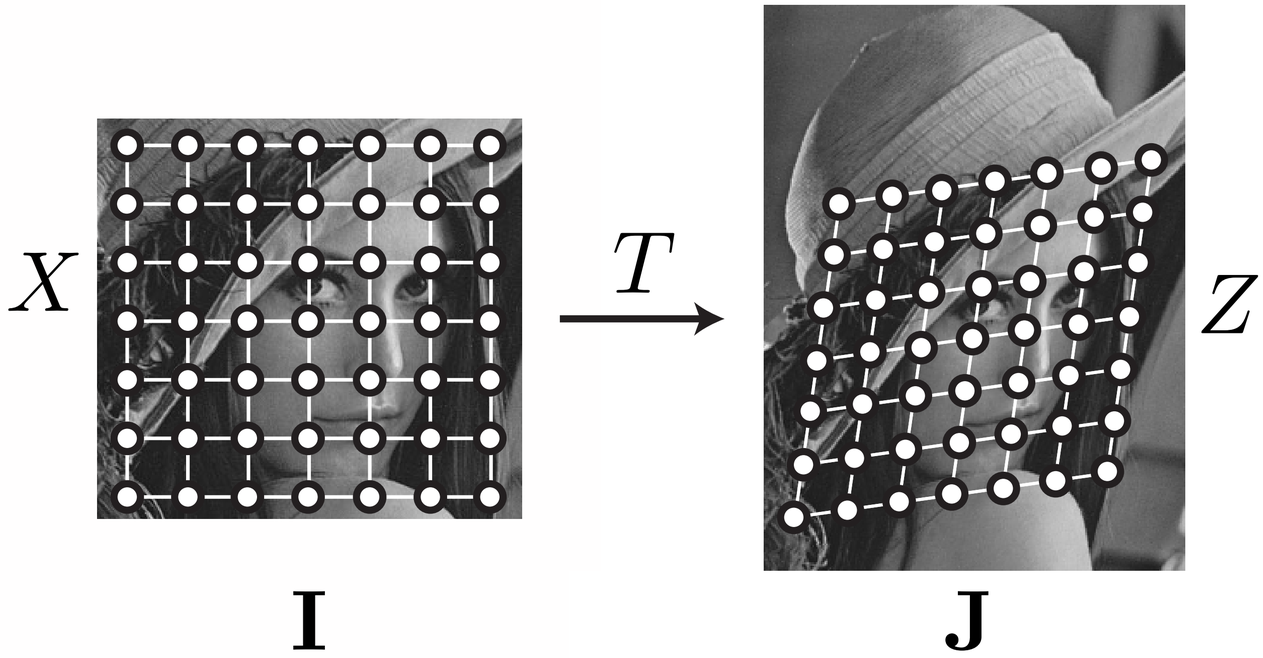
?



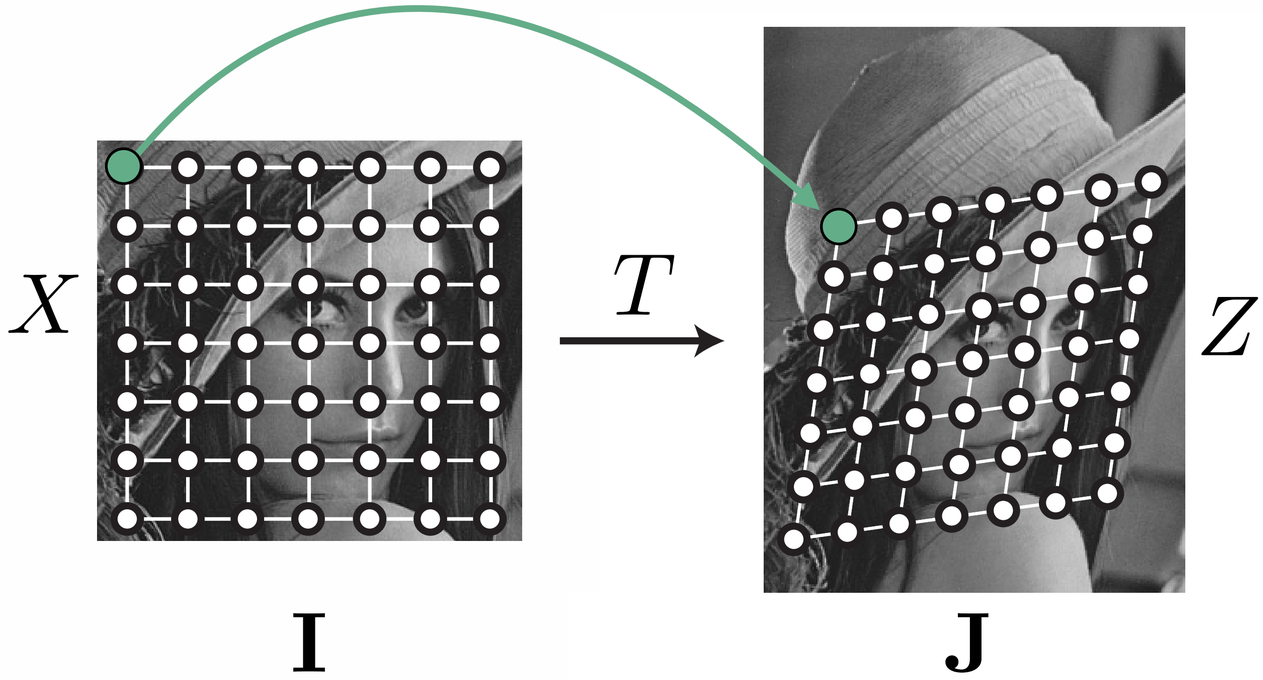
I

J

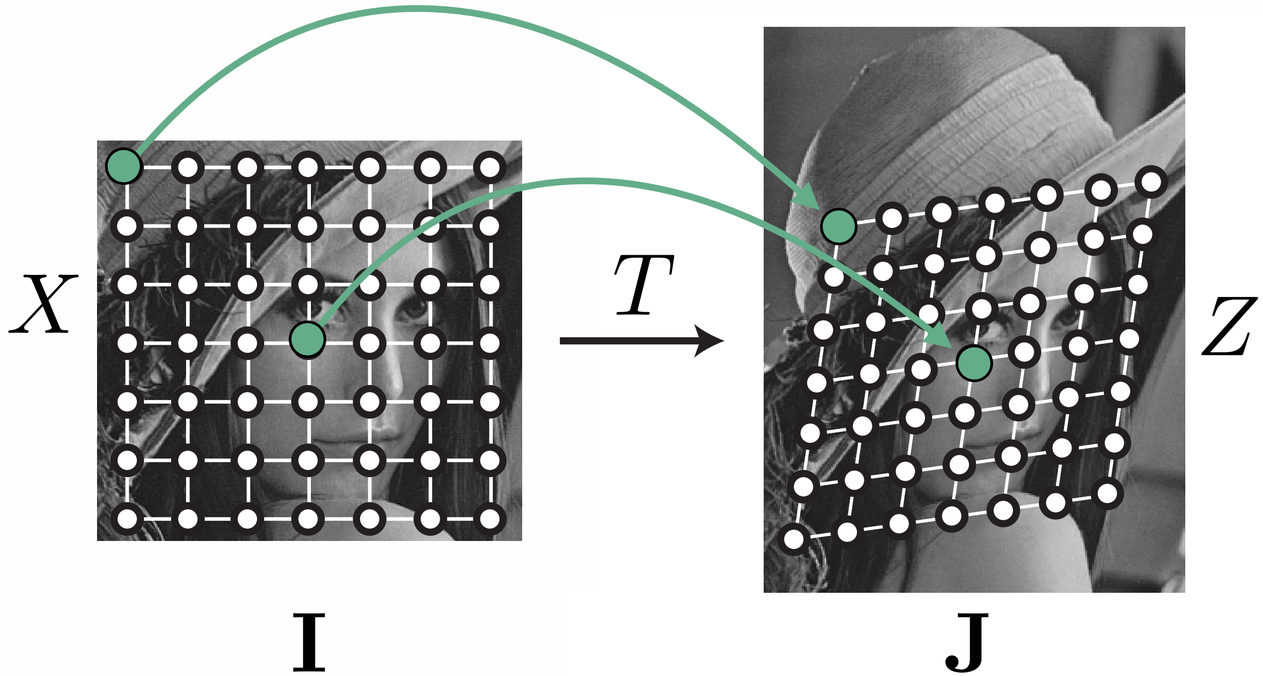
2D Registration



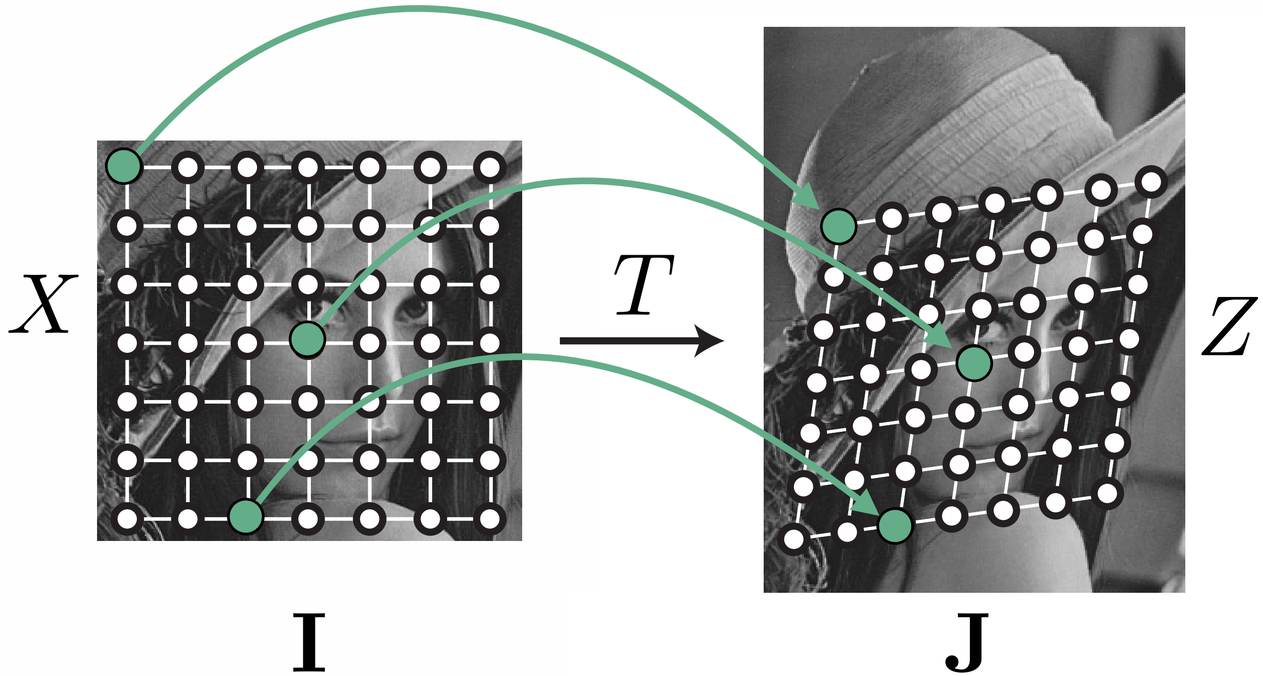
2D Registration



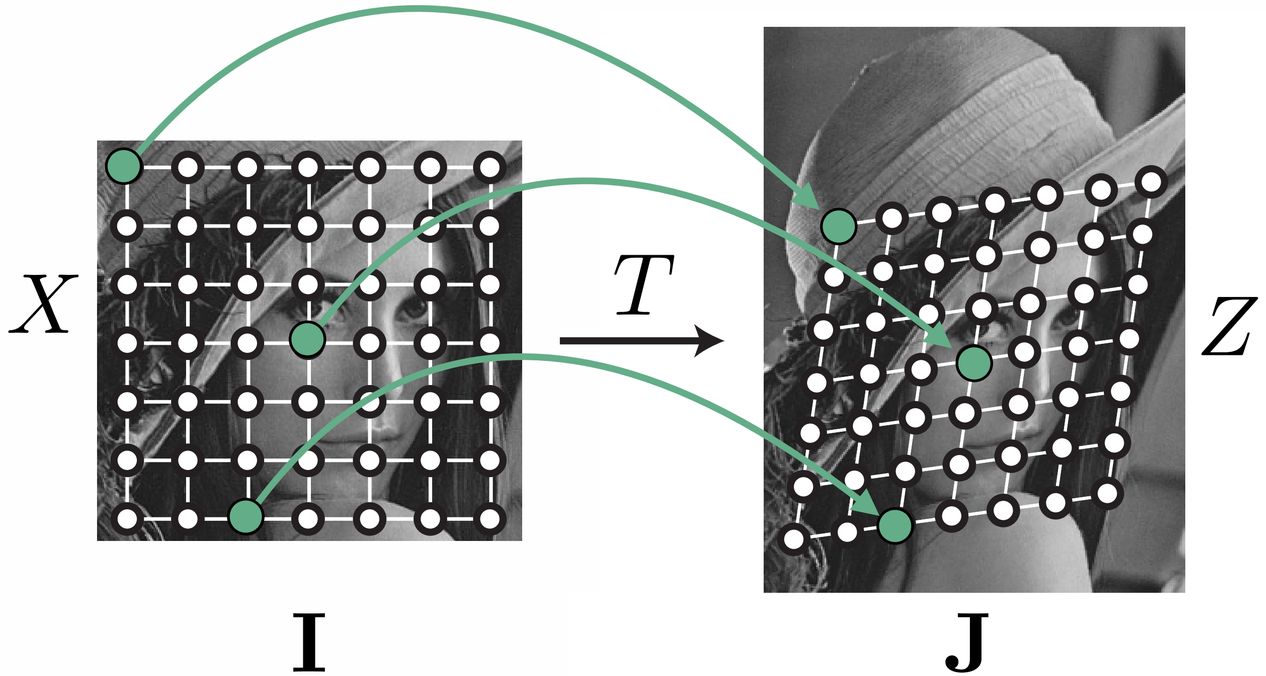
2D Registration



2D Registration

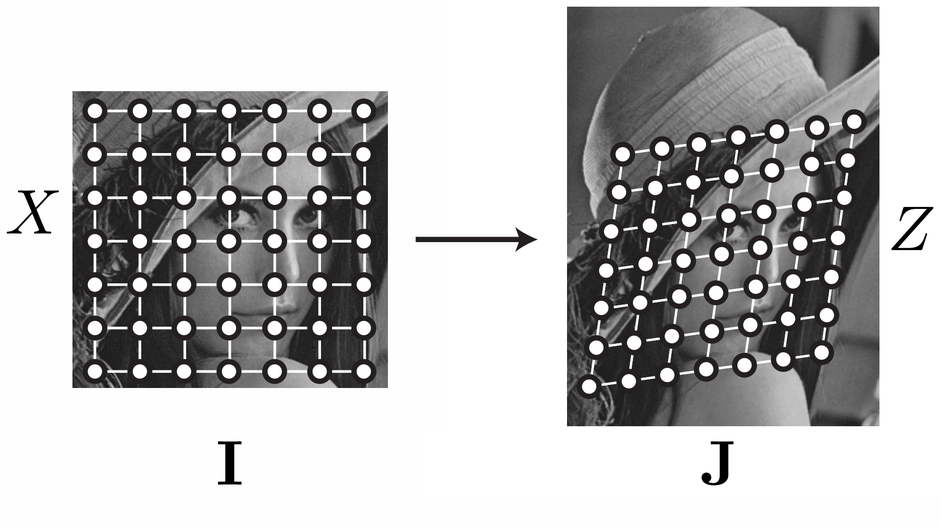


2D Registration



DEMO

2D Registration - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{z}_i)\|_2^2$$

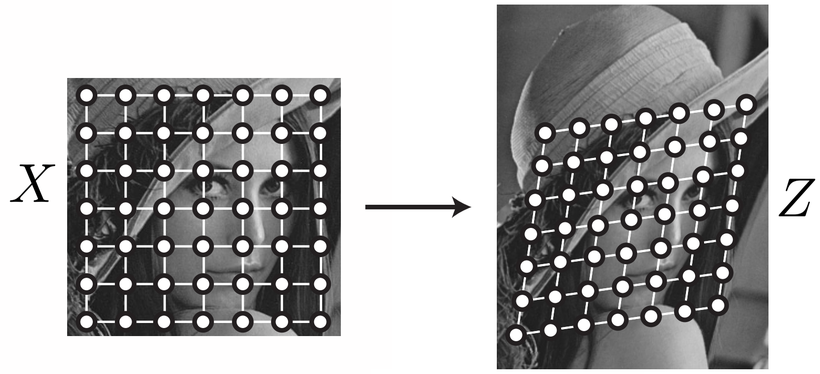
color value at $\mathbf{x}_i$ of image $\mathbf{I}$

color value at $\mathbf{z}_i$ of image $\mathbf{J}$

2D Registration - Prior

- Lucas-Kanade
  - Constant displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

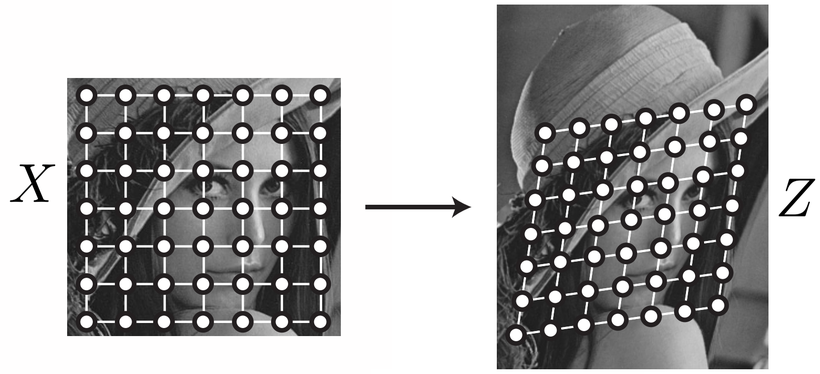


$$E_{\text{prior}}(Z) = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{x}_j) - (\mathbf{z}_i - \mathbf{x}_i)\|_2^2$$

2D Registration - Prior

- Lucas-Kanade
  - Constant displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



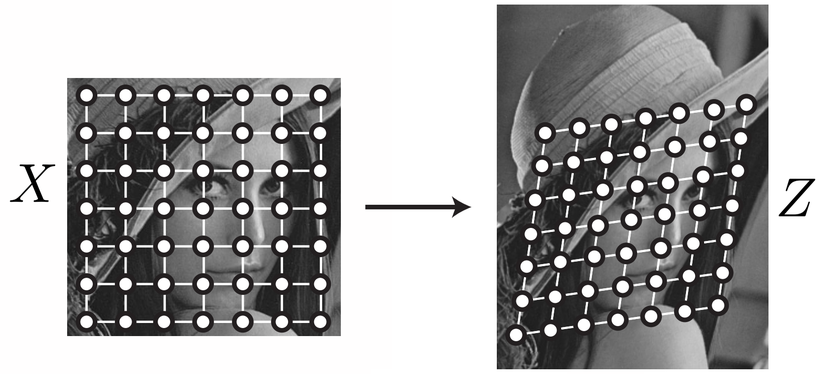
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“vertex” wants...

2D Registration - Prior

- Lucas-Kanade
  - Constant displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



...its deformation...

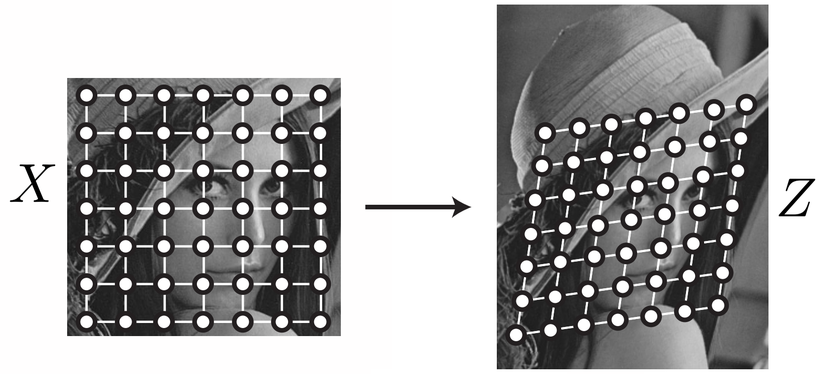
$$E_{\text{prior}}(Z) = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \left\| (\mathbf{z}_j - \mathbf{x}_j) - (\mathbf{z}_i - \mathbf{x}_i) \right\|_2^2$$

"vertex" wants...

2D Registration - Prior

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...its deformation...

$$E_{\text{prior}}(Z) = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \left\| \left(\mathbf{z}_j - \mathbf{x}_j\right) - \left(\mathbf{z}_i - \mathbf{x}_i\right) \right\|_2^2$$

“vertex” wants...

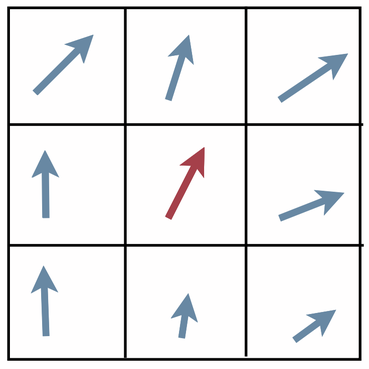
...to match its neighbours’

2D Registration - Prior

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$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

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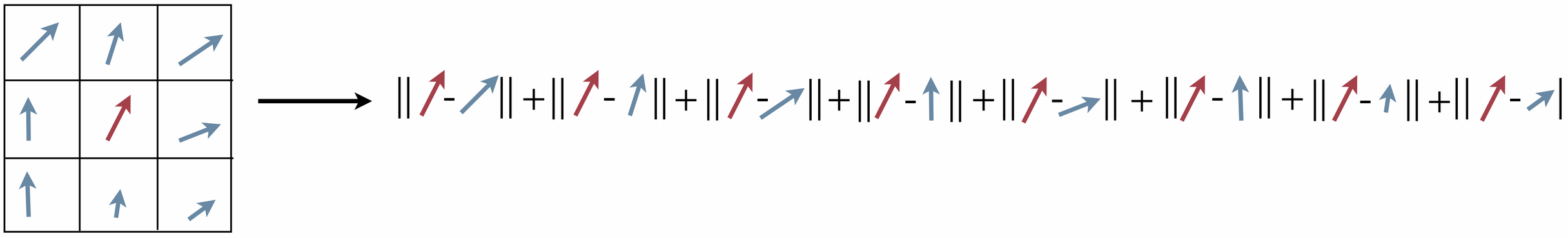
Lucas, Kanade: **An Iterative Image Registration Technique with an Application to Stereo Vision**, *IJCAI 1981*

2D Registration - Prior

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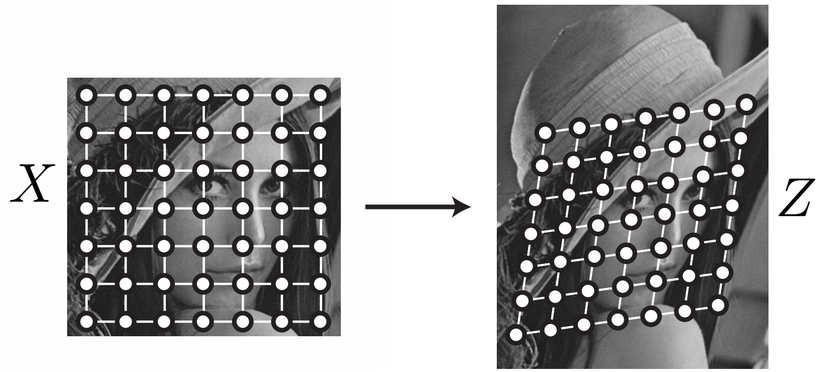


Lucas, Kanade: **An Iterative Image Registration Technique with an Application to Stereo Vision**, *IJCAI 1981*

2D Registration - Prior

- Horn-Schunck
  - Smooth displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



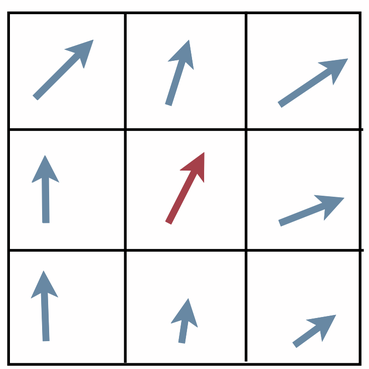
$$E_{\text{prior}}(Z) = \sum_{i=1}^{n} \left\| (\mathbf{z}_i - \mathbf{x}_i) - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (\mathbf{z}_j - \mathbf{x}_j) \right\|_2^2$$

2D Registration - Prior

- Horn-Schunck
  - Smooth displacement field

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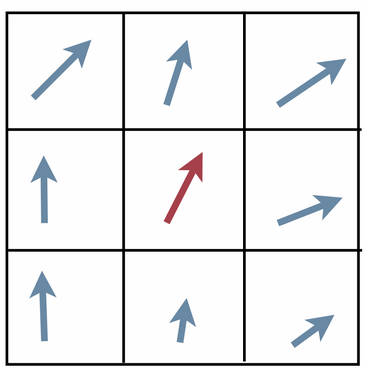
Horn, Schunk: "**Determining Optical Flow**", *Artificial Intelligence Journal 1981*

2D Registration - Prior

- Horn-Schunck
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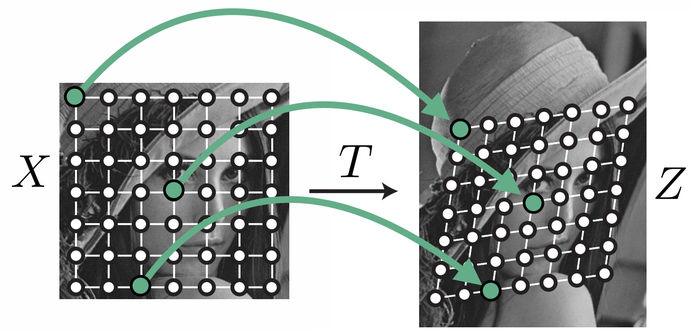


Horn, Schunk: "**Determining Optical Flow**", *Artificial Intelligence Journal 1981*

2D Registration - Recap

2D Registration - Recap

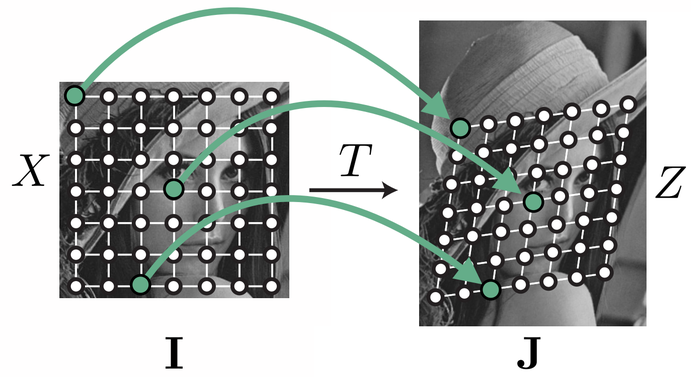
Matching



$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{z}_i)\|_2^2$$

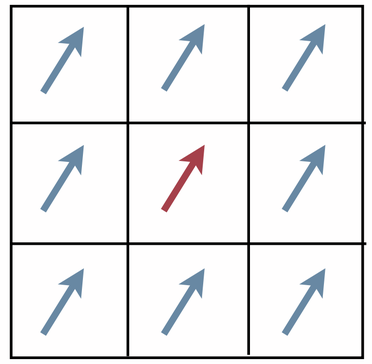
2D Registration - Recap

Matching

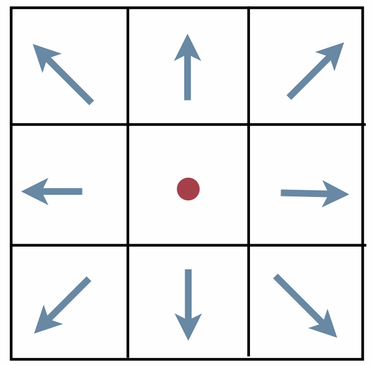


$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{z}_i)\|_2^2$$

Prior



constant (Lucas-Kanade)



smooth (Horn-Schunck)

Overview

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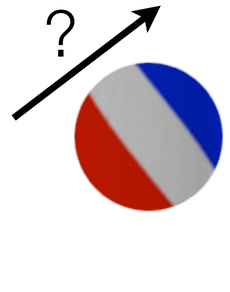
Conclusions

Q&A

2D/3D Registration



2D/3D Registration



2D/3D Registration

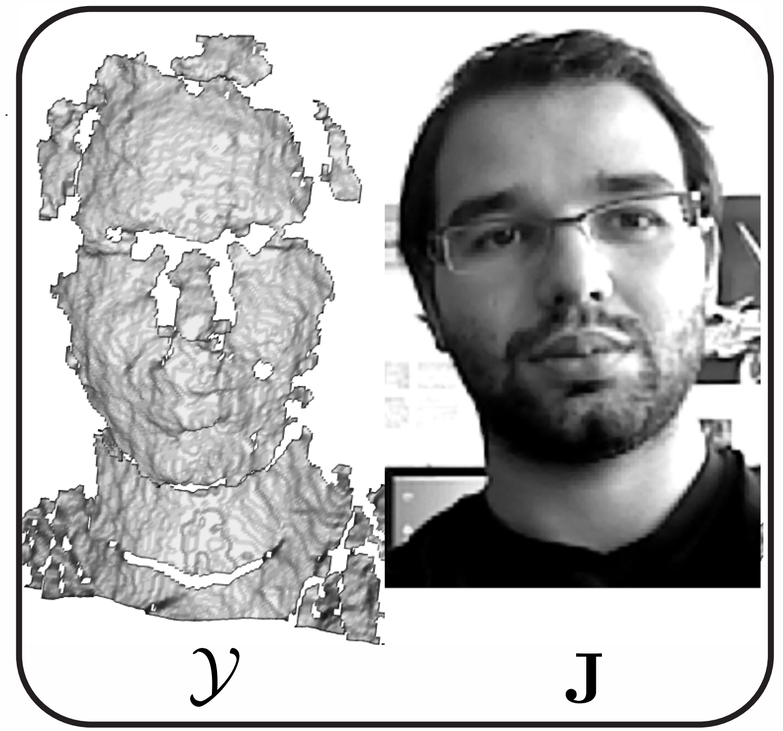


2D/3D Registration



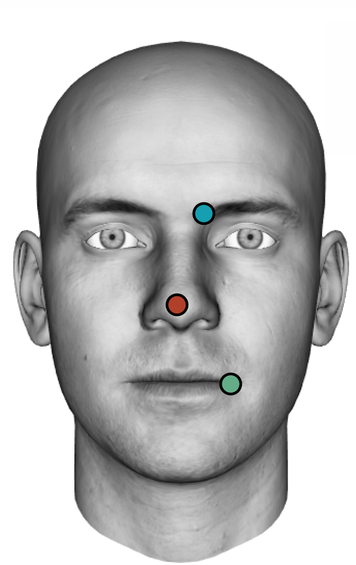
$\mathcal{X}$,$\mathbf{I}$

?



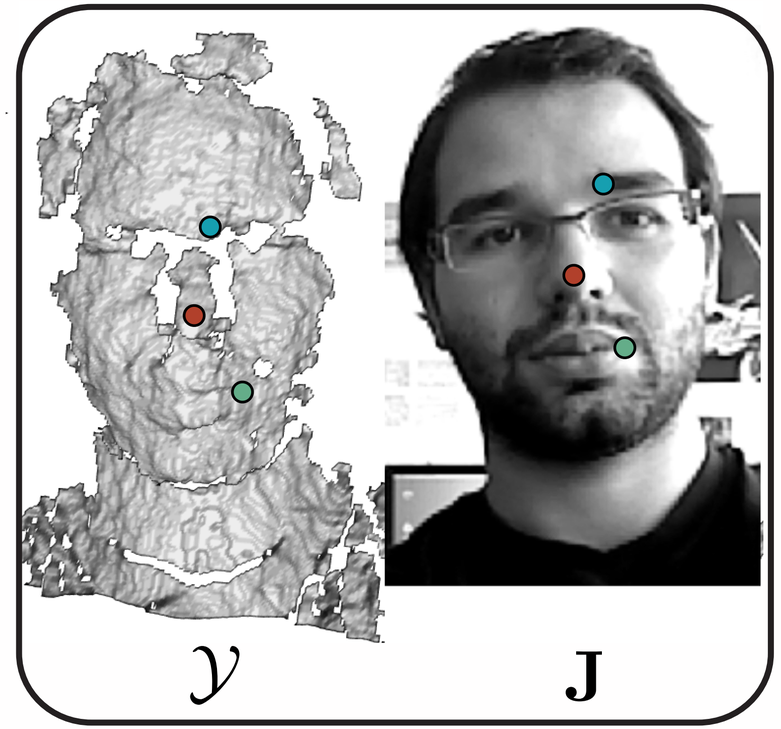
$\mathcal{Y}$ $\mathbf{J}$

2D/3D Registration



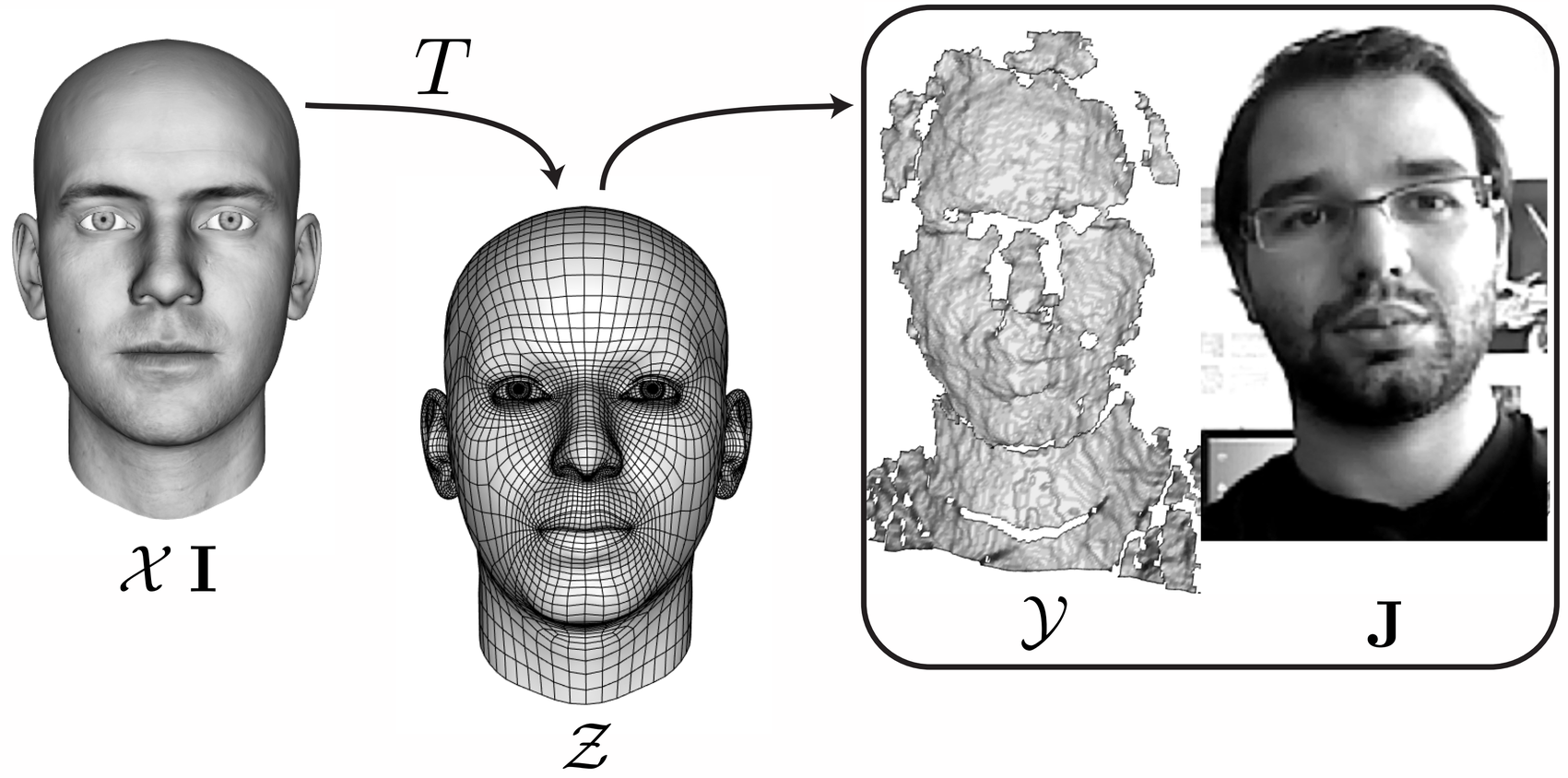
$\mathcal{X}$,$\mathbf{I}$

?

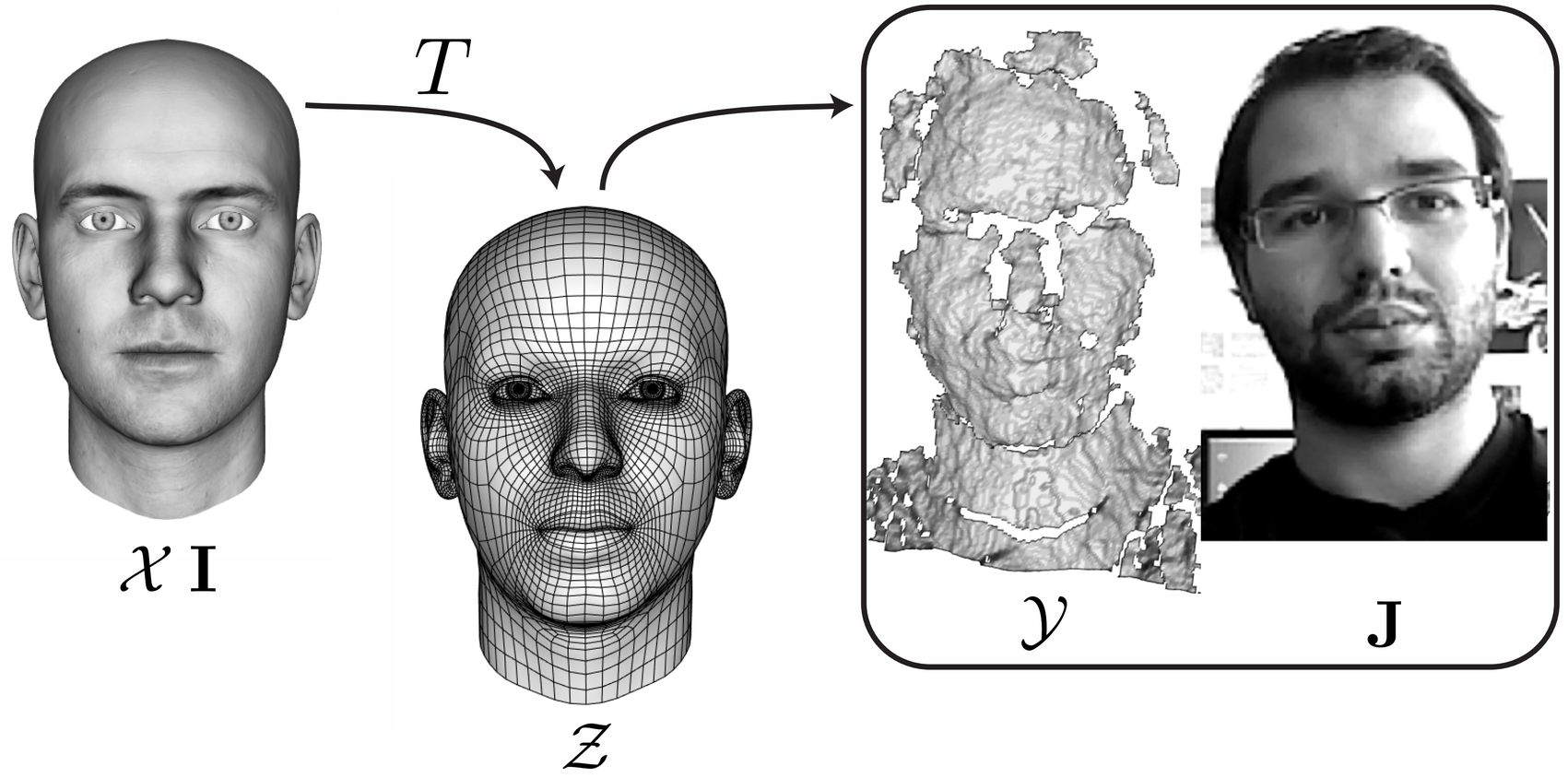


$\mathcal{Y}$ $\mathbf{J}$

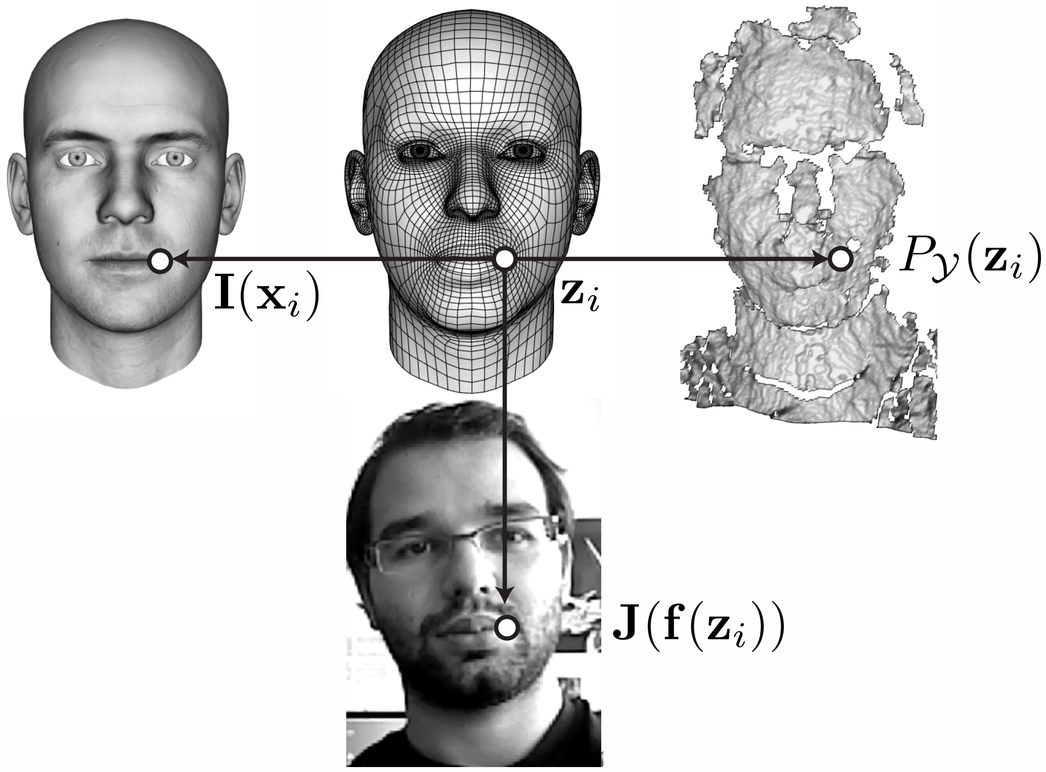
2D/3D Registration



2D/3D Registration



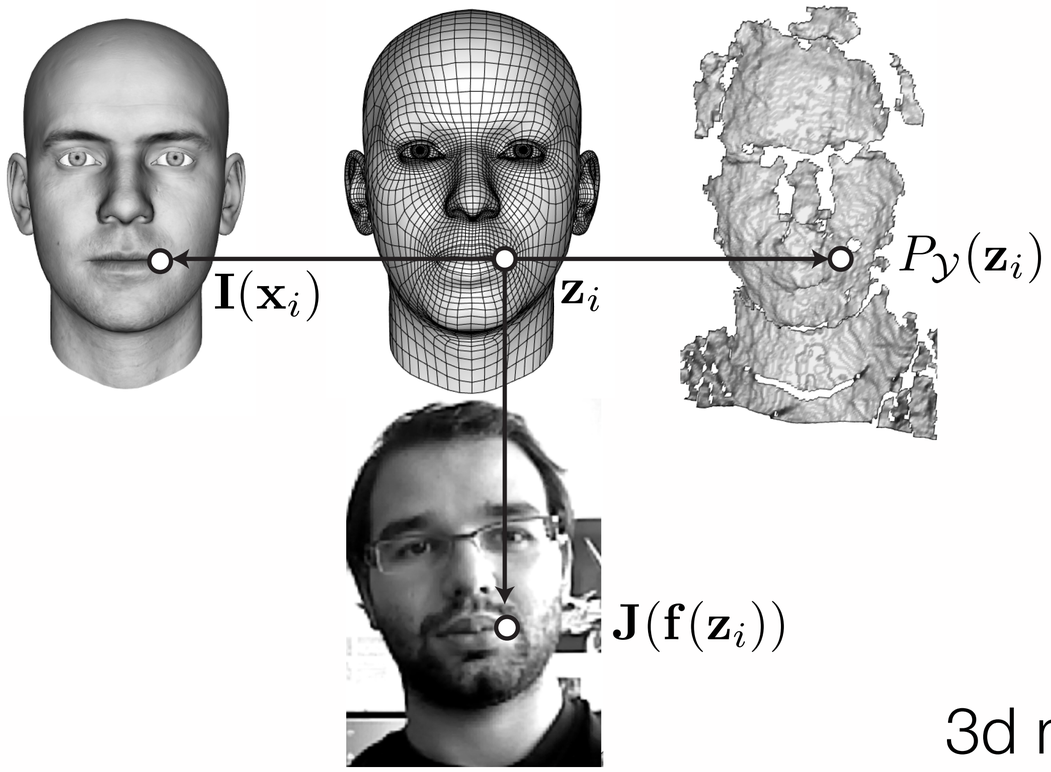
2D/3D - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(Z) = w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

2D/3D - Matching

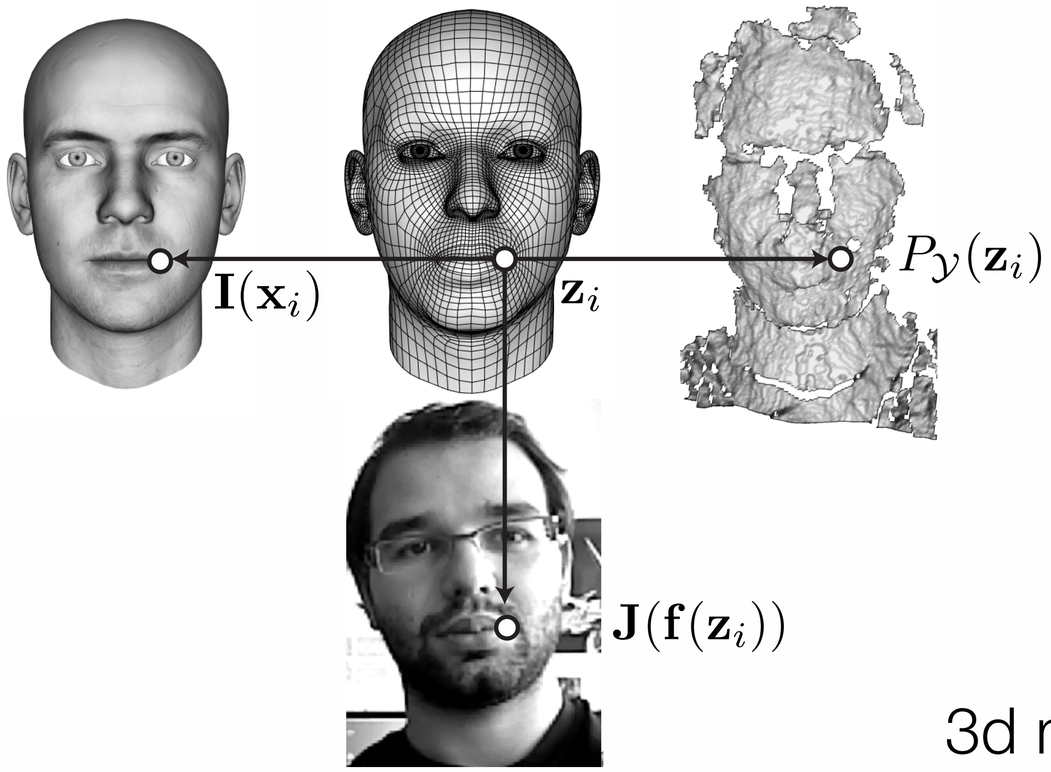


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3d matching

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2D/3D - Matching



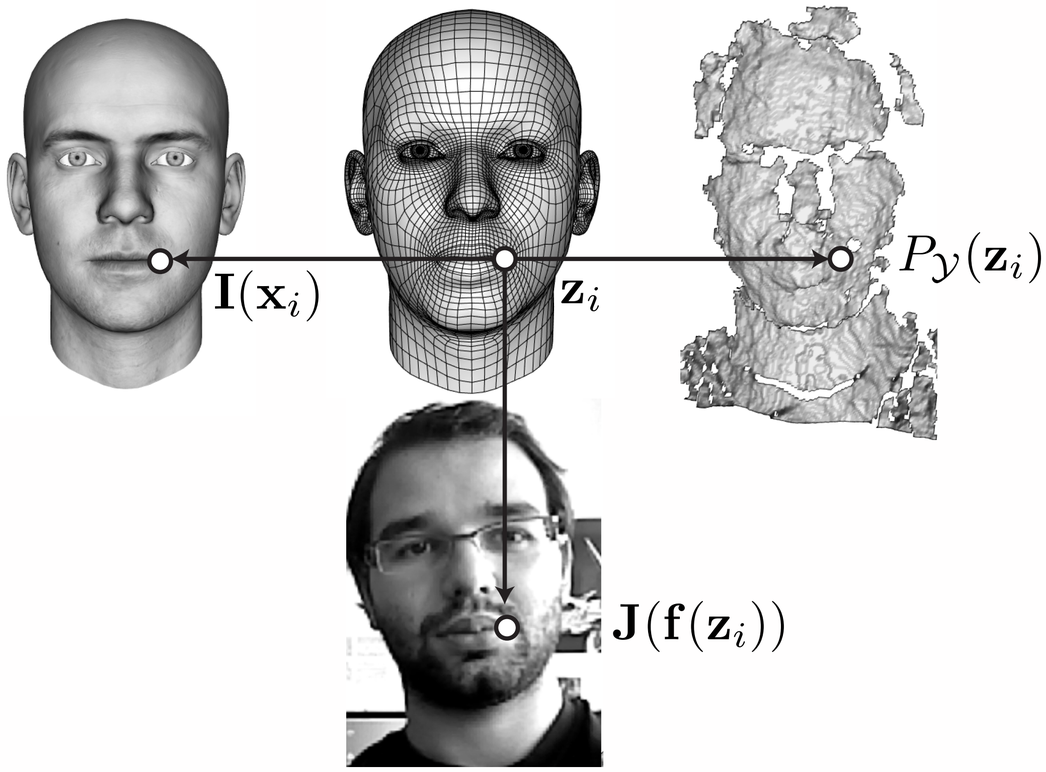
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3d matching

2d matching

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2D/3D - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

projection of the 3D vertex in the 2D image

$$E_{\text{match}}(Z) = w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

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Orthographic projection $\mathbf{f}(\mathbf{z}_i) = \begin{bmatrix} \mathbf{z}_{i,x} & \mathbf{z}_{i,y} \end{bmatrix}^T$

2D/3D - Matching

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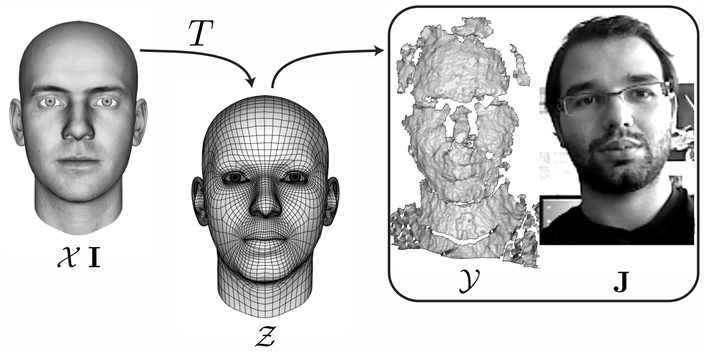
Orthographic projection $\mathbf{f}(\mathbf{z}_i) = \begin{bmatrix} \mathbf{z}_{i,x} & \mathbf{z}_{i,y} \end{bmatrix}^T$

Perspective projection $\mathbf{f}(\mathbf{z}_i) = \begin{bmatrix} \frac{f\mathbf{z}_{i,x}}{\mathbf{z}_{i,z}} & \frac{f\mathbf{z}_{i,y}}{\mathbf{z}_{i,z}} \end{bmatrix}^T$

2D/3D Registration - Recap

2D/3D Registration - Recap

Matching



$$E_{\text{match}}(Z) = w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

$$+ w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

Overview

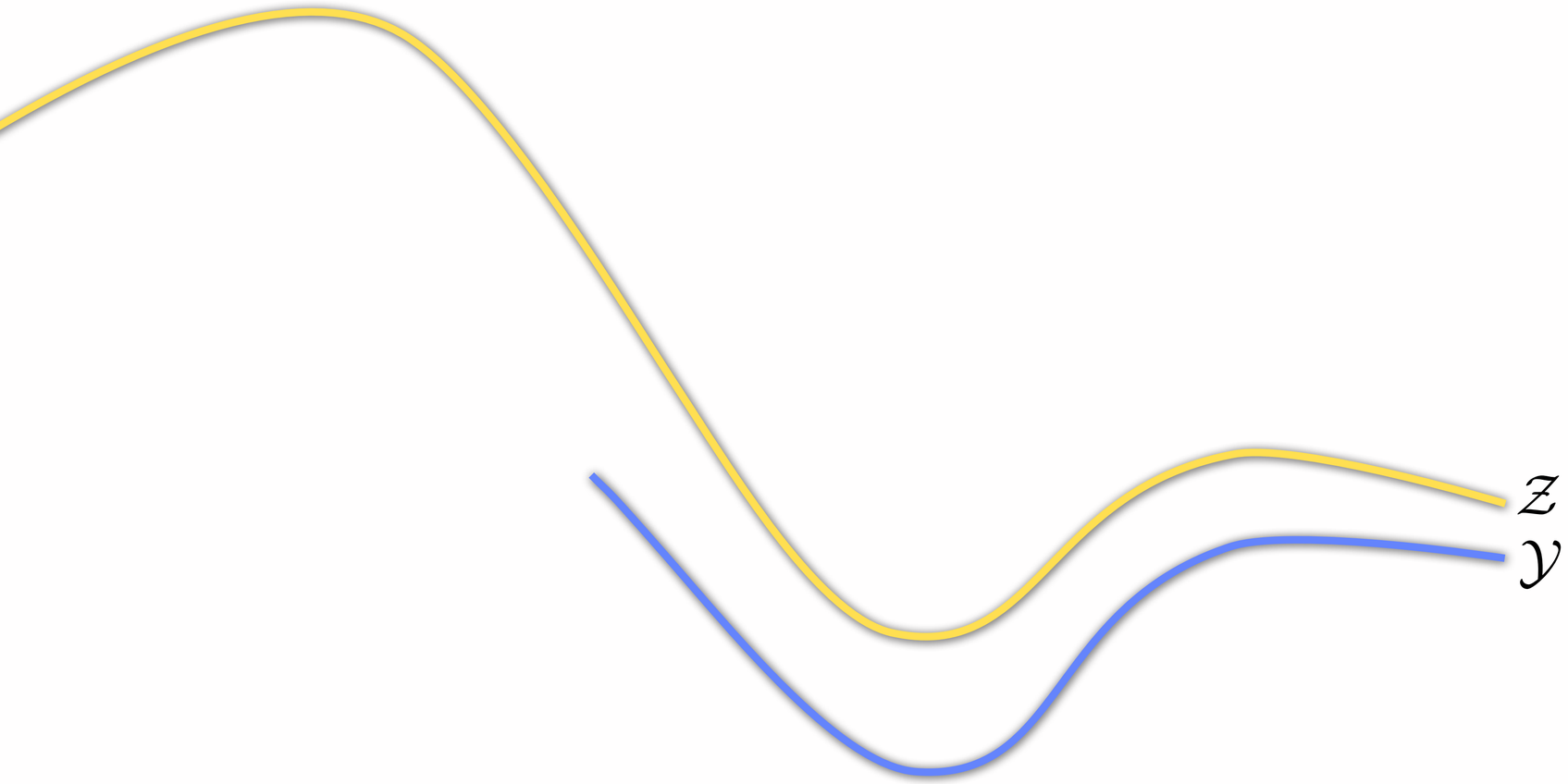
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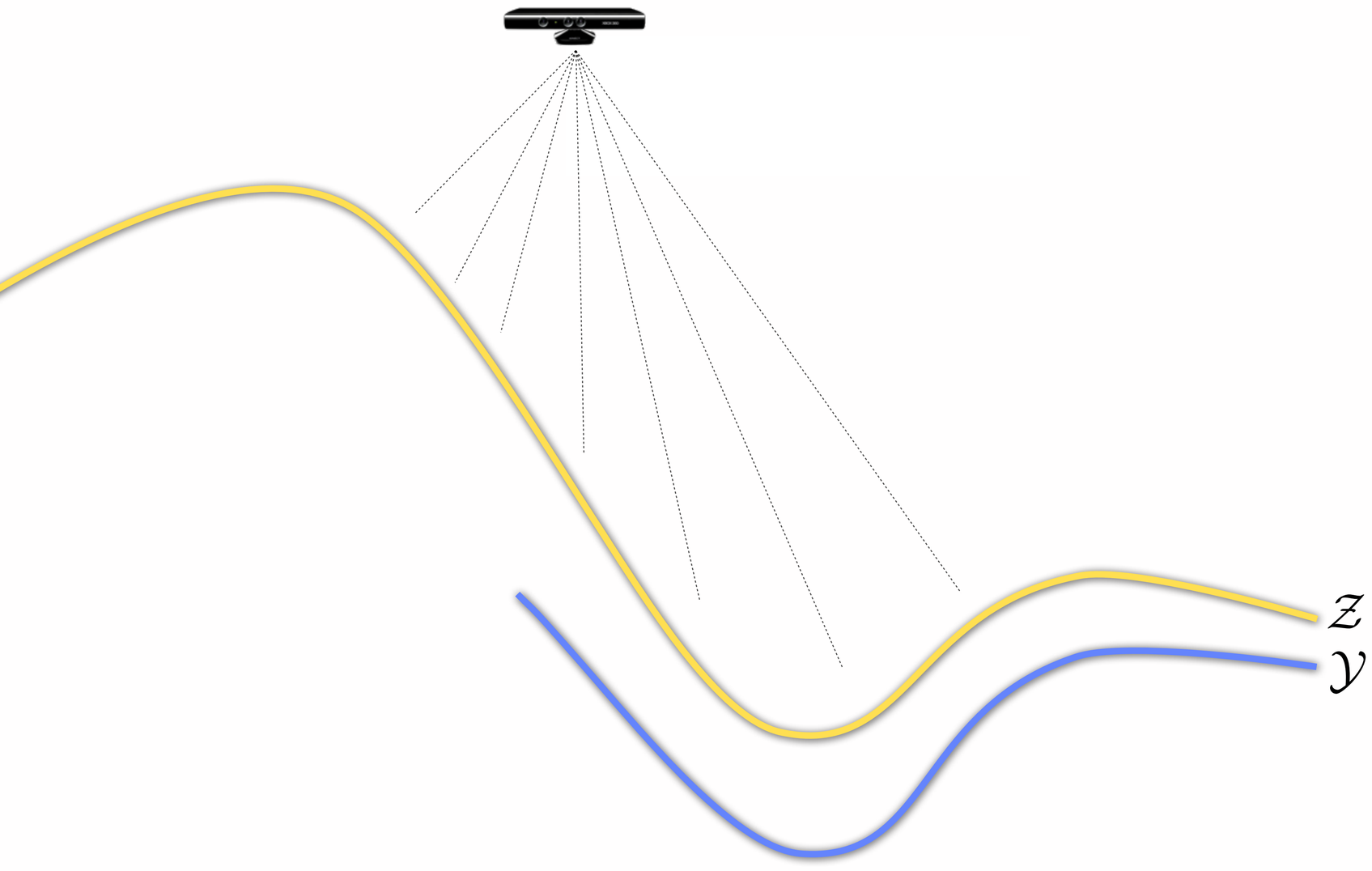
Conclusions

Q&A

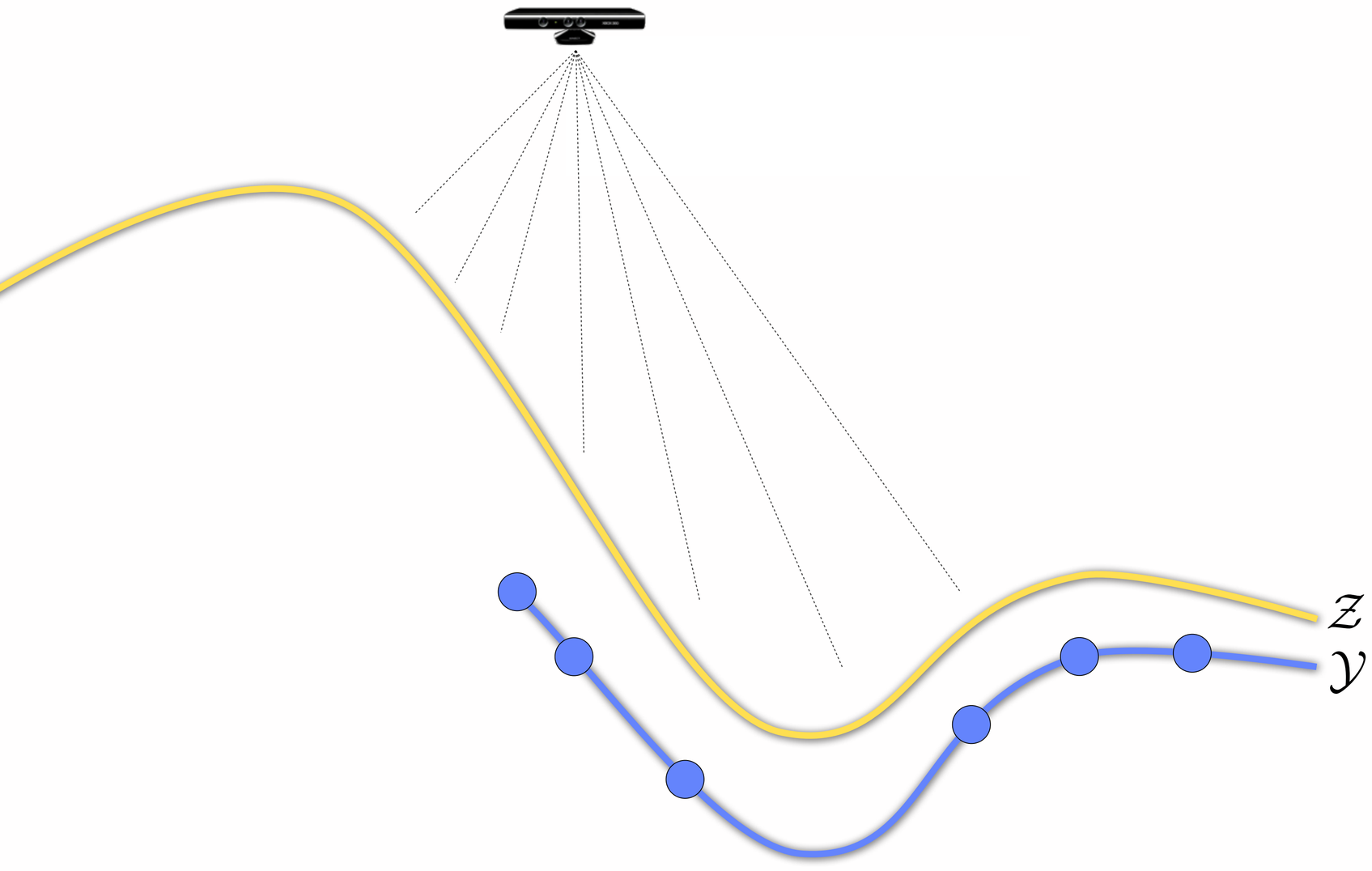
Robust Registration



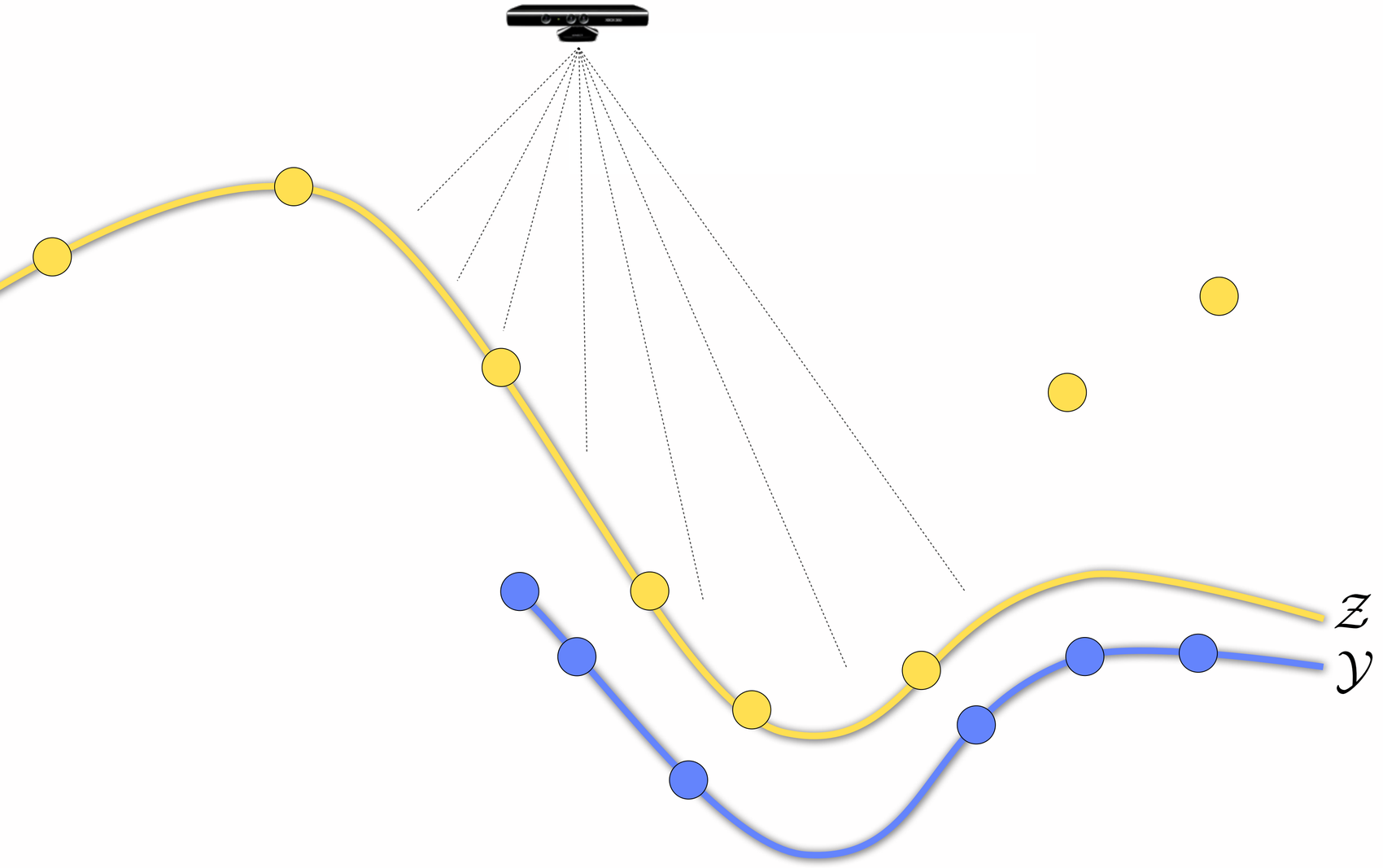
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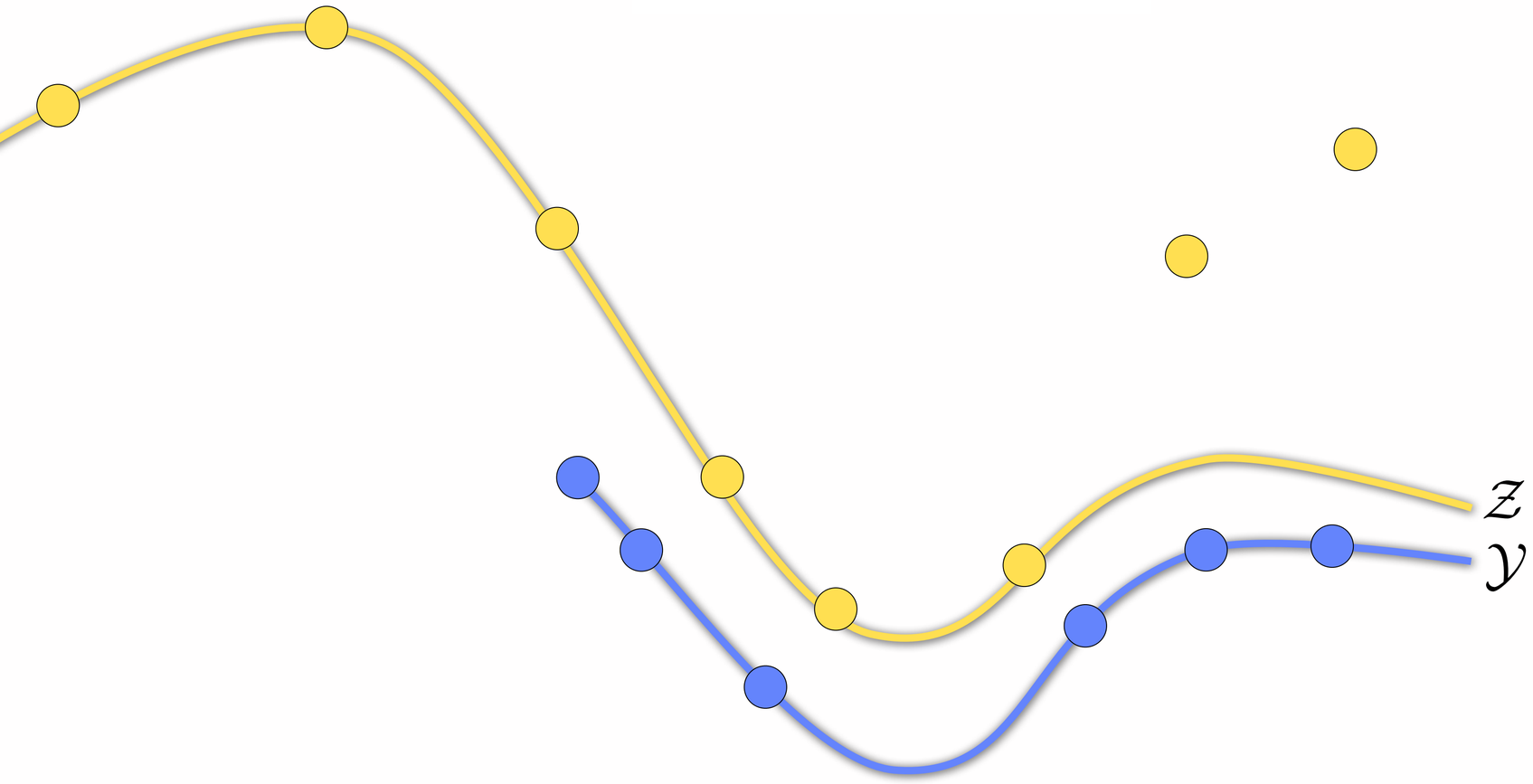
Robust Registration



Robust Registration

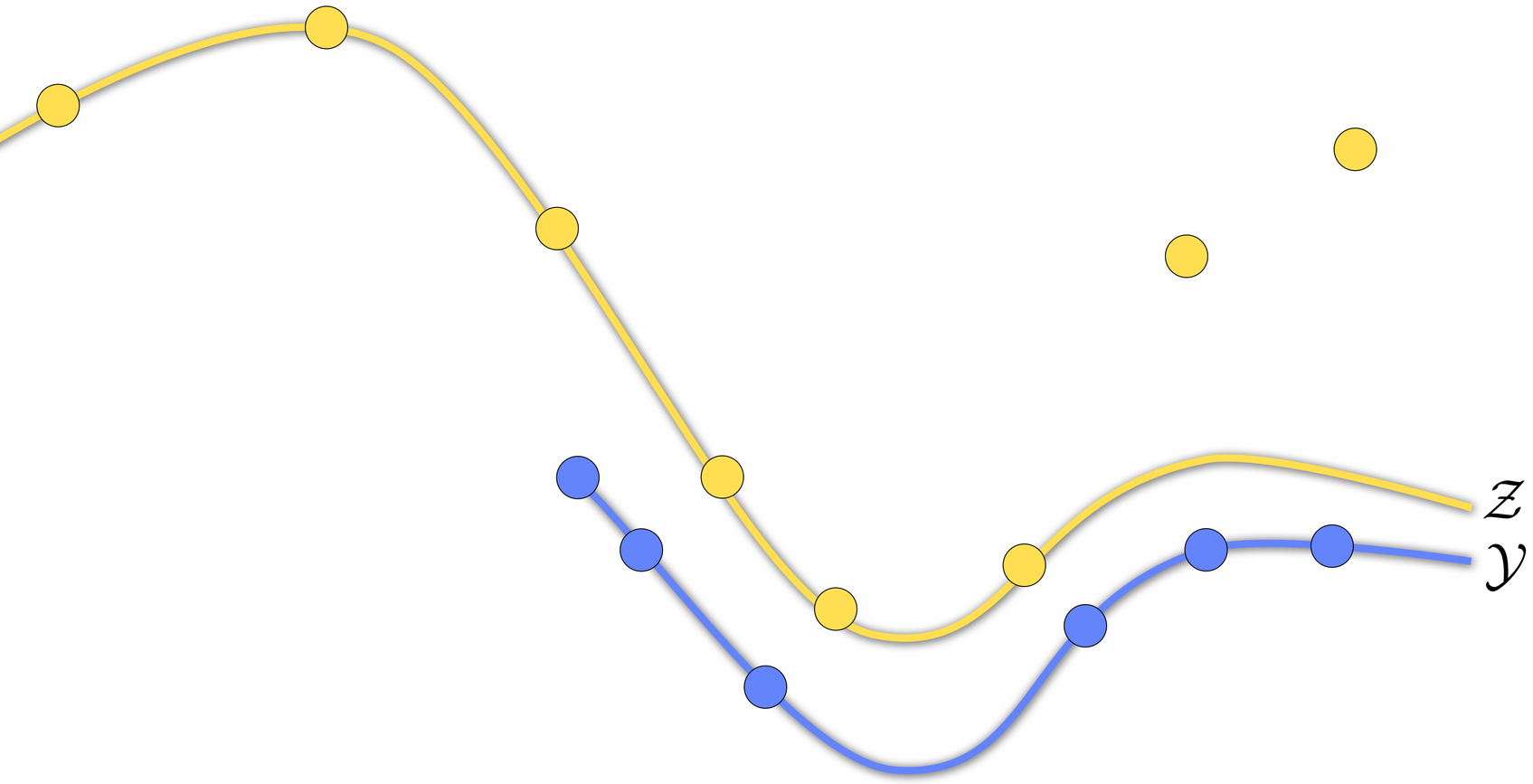


Robust Registration



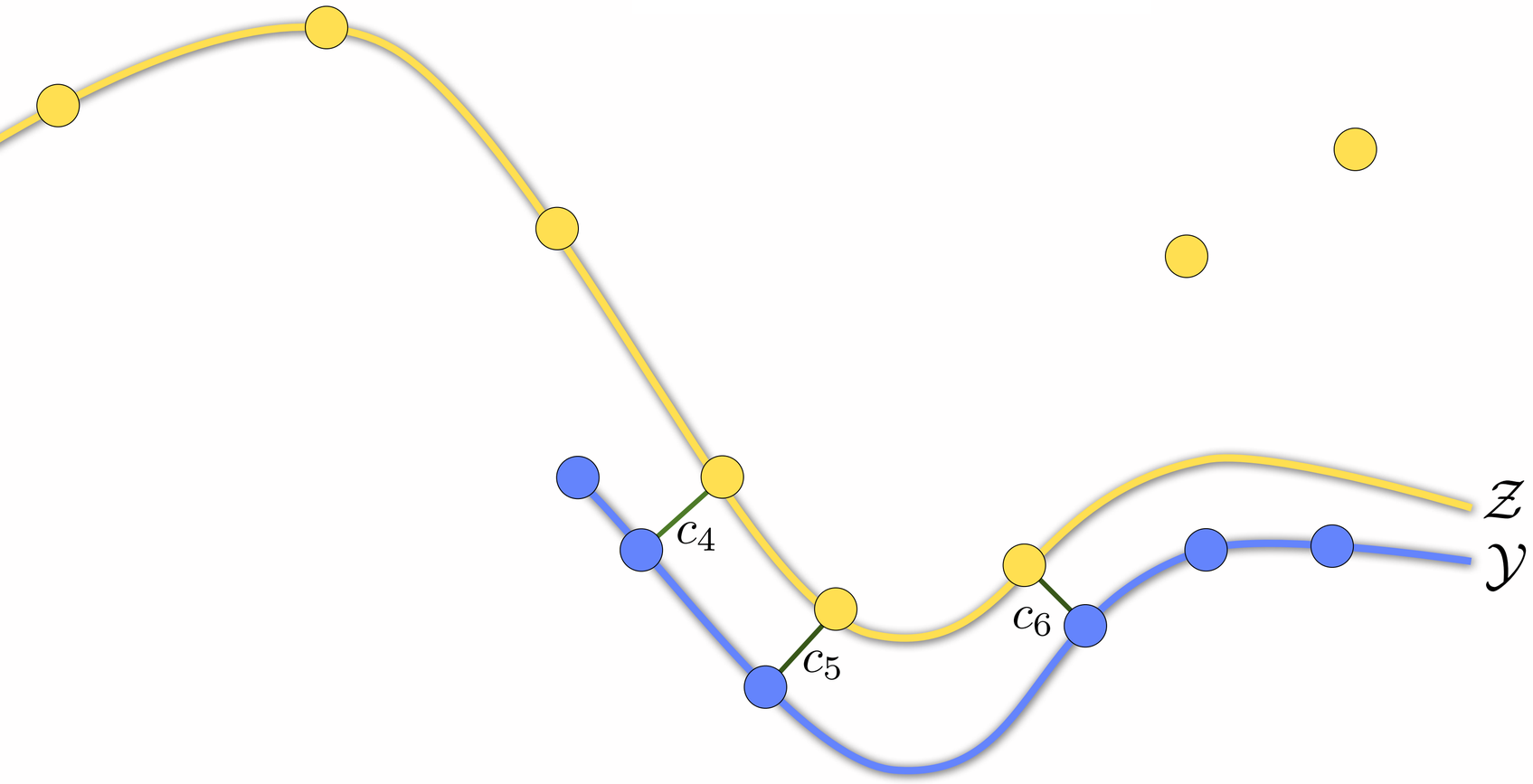
Robust Registration

$$c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$



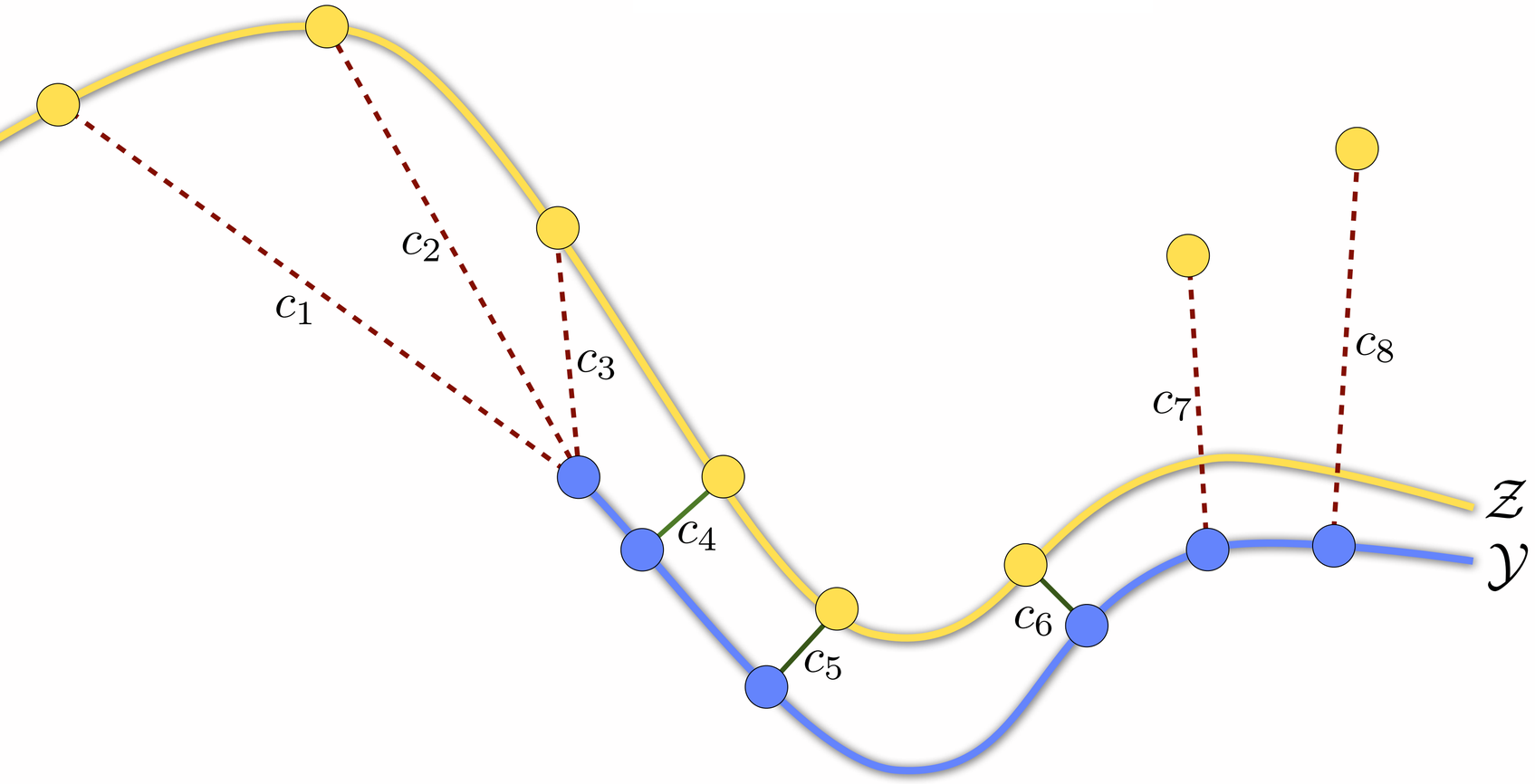
Robust Registration

$$c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$



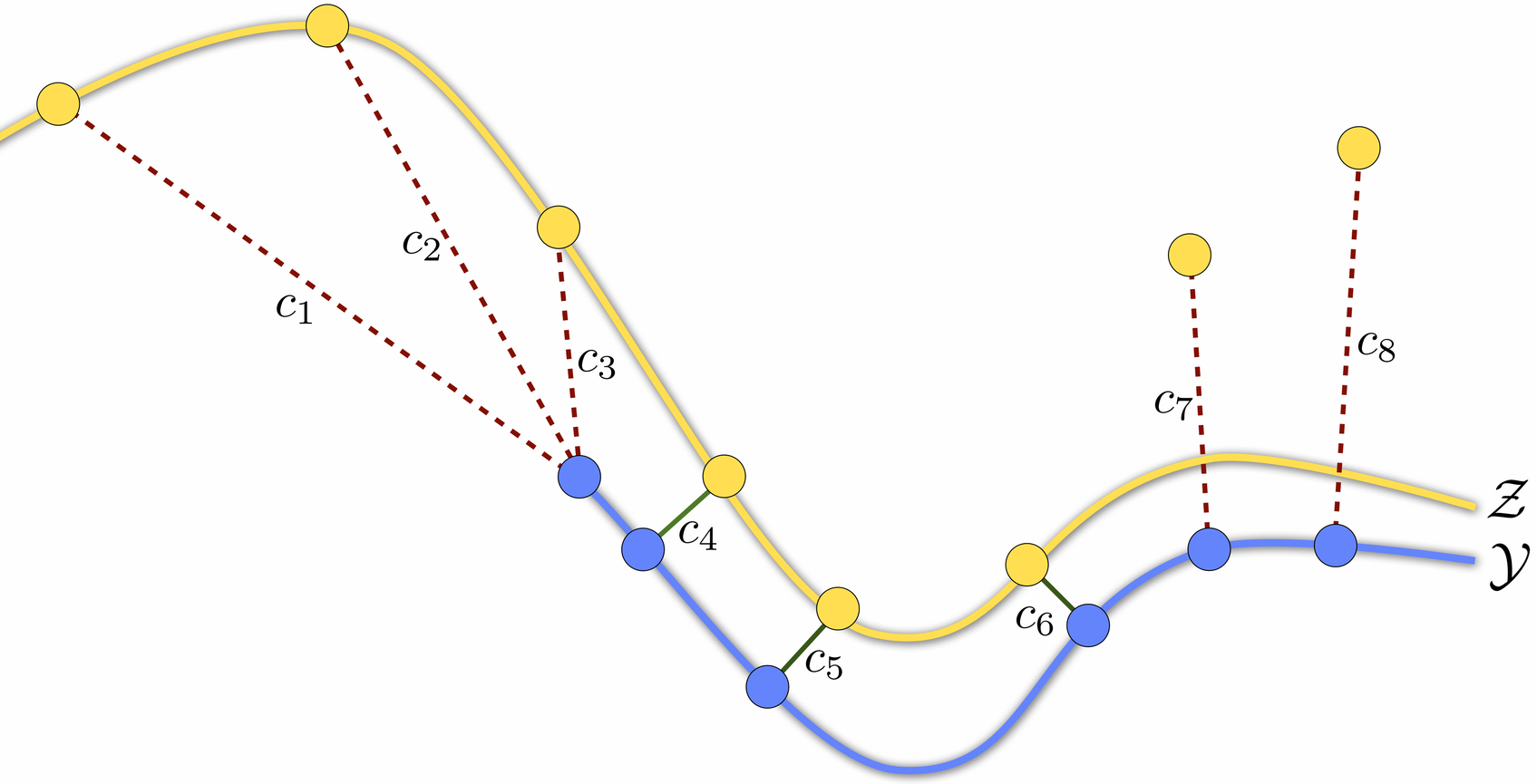
Robust Registration

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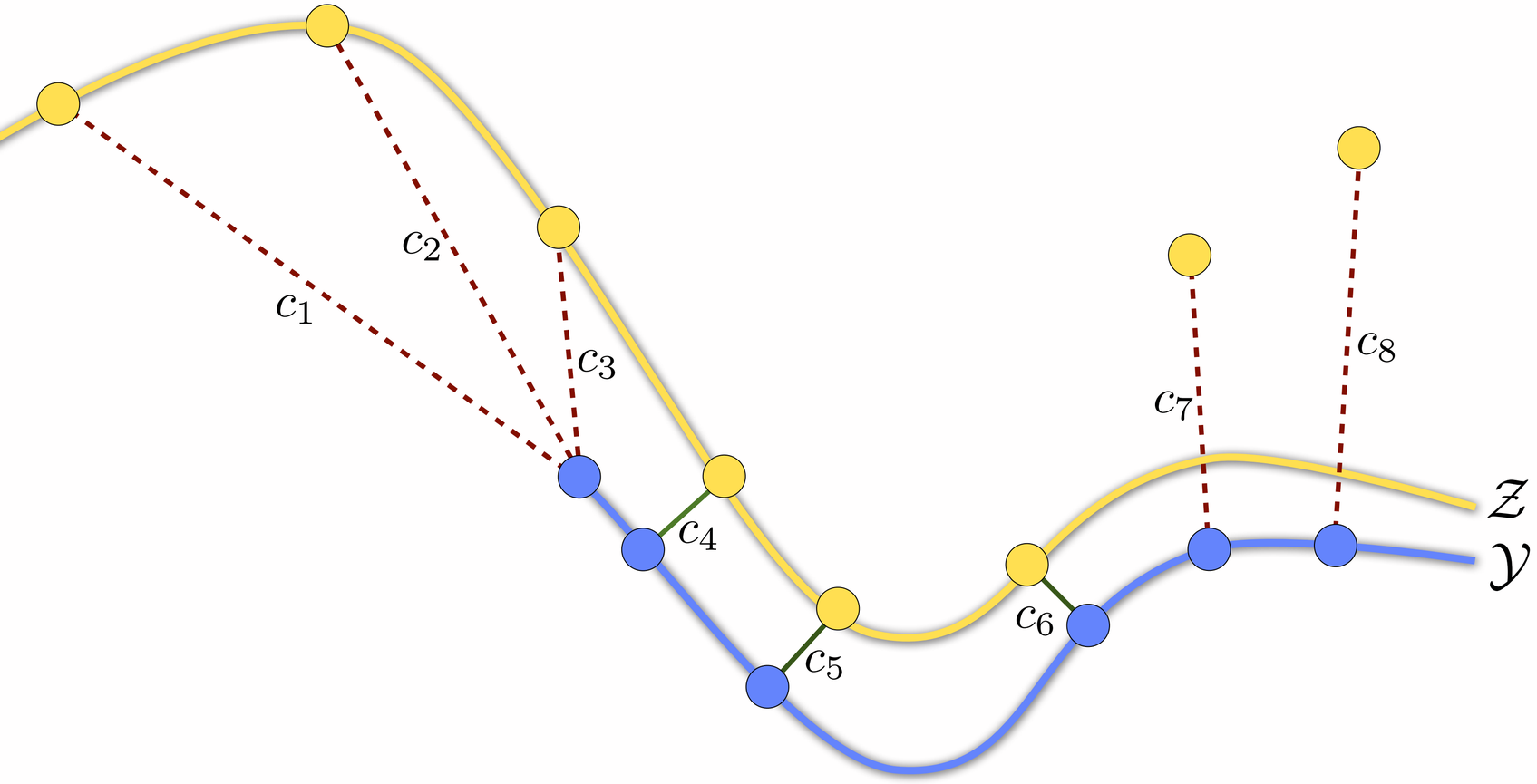
Robust Registration

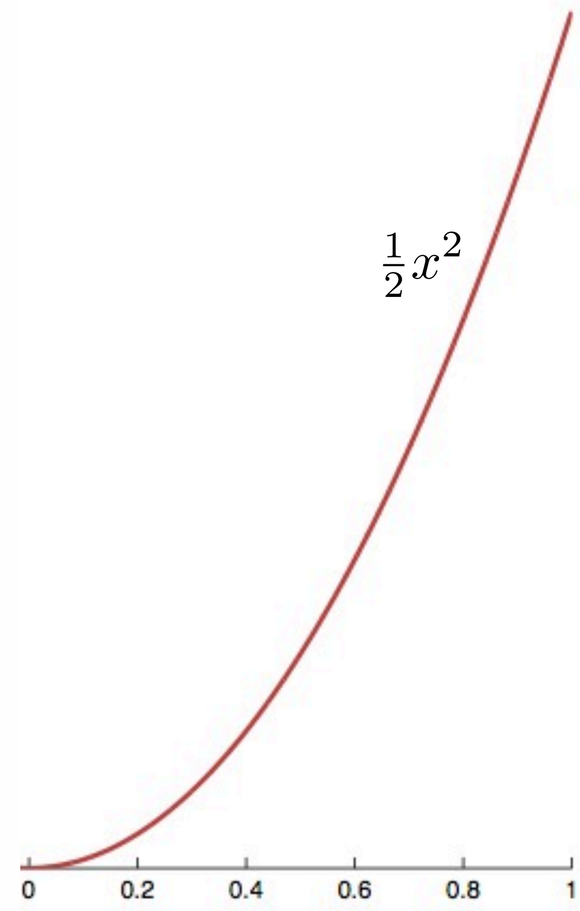
$$E_{\text{match}}(Z) = \sum_{i=1}^{n} c_i^2, \quad c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$



Robust Registration

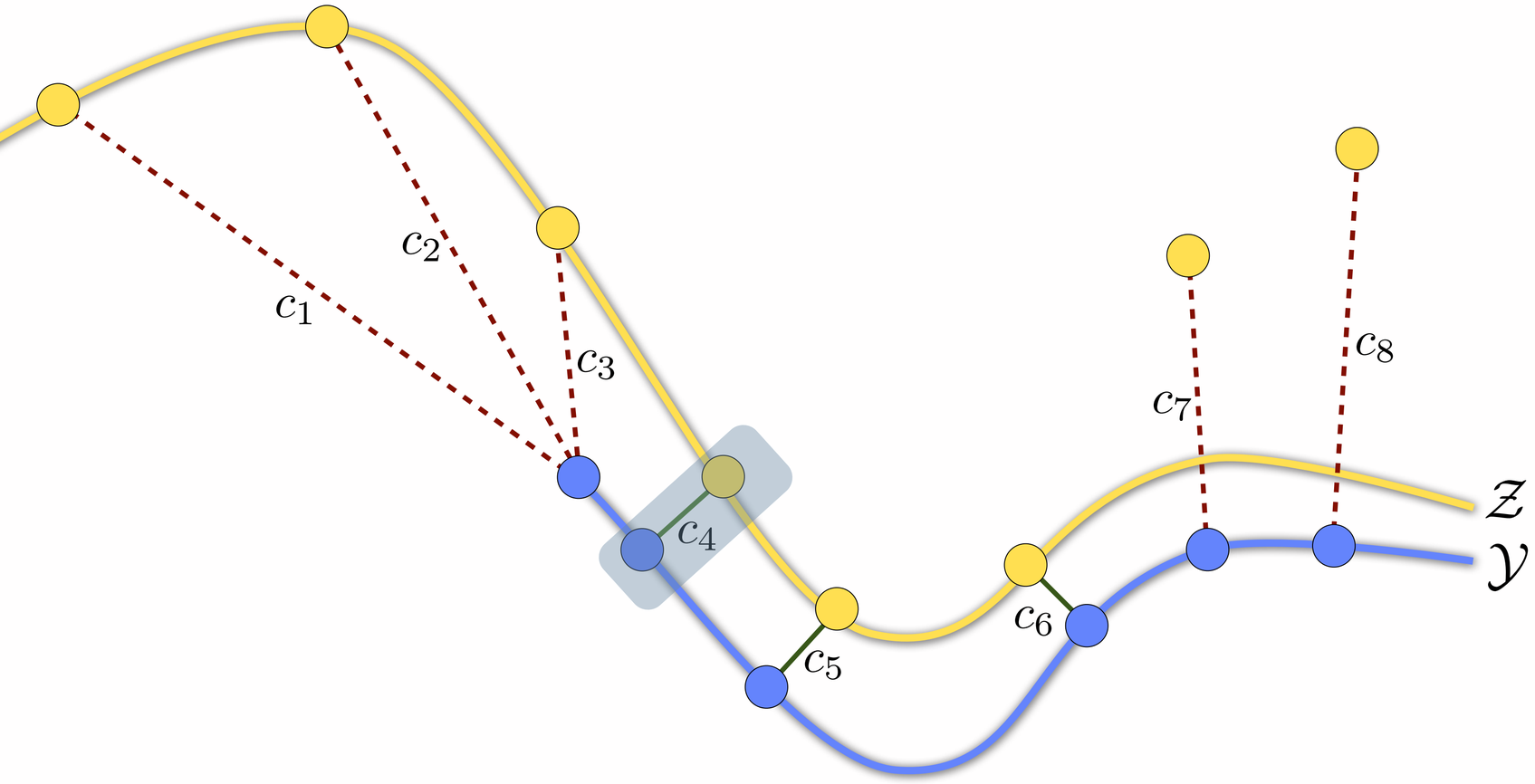
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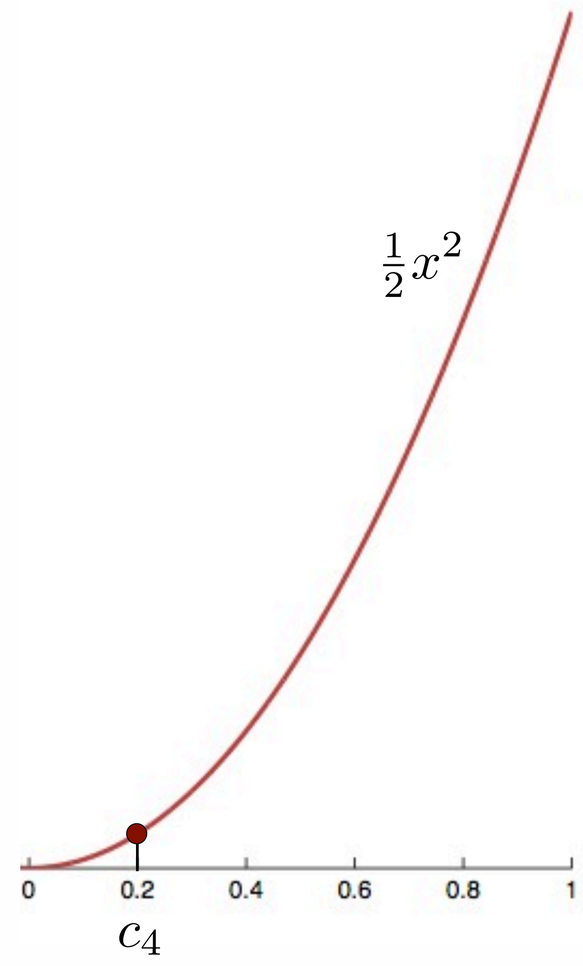




Robust Registration

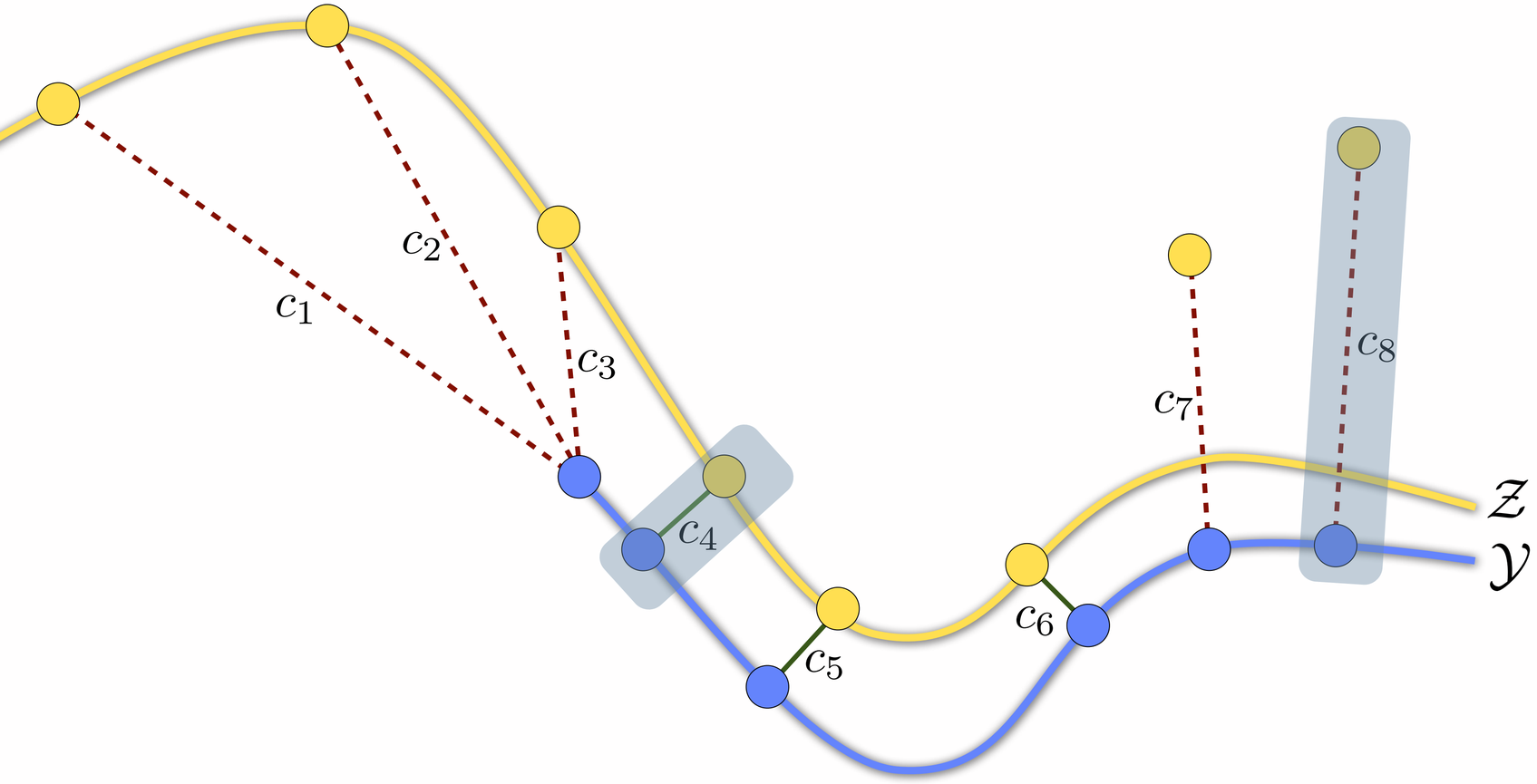
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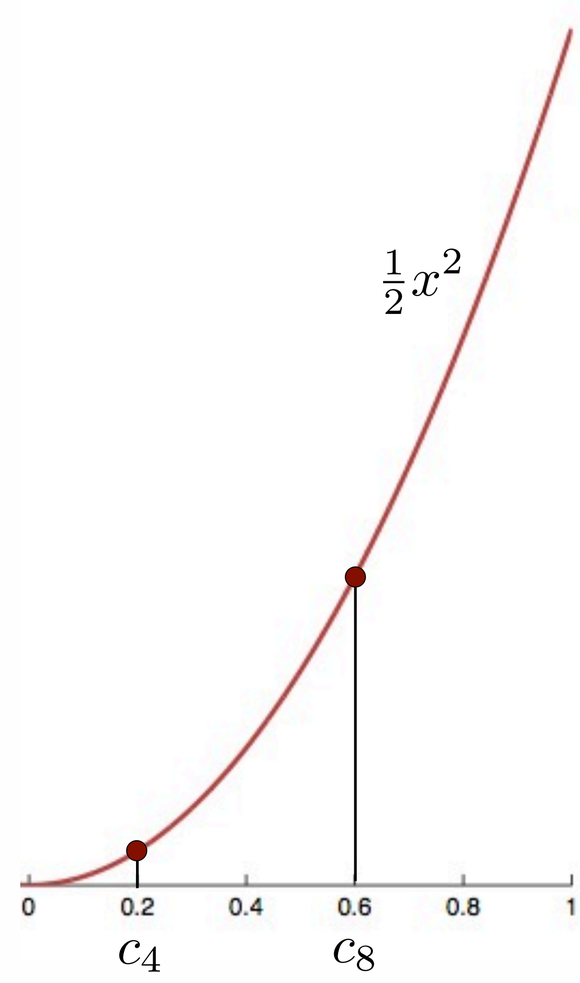




Robust Registration

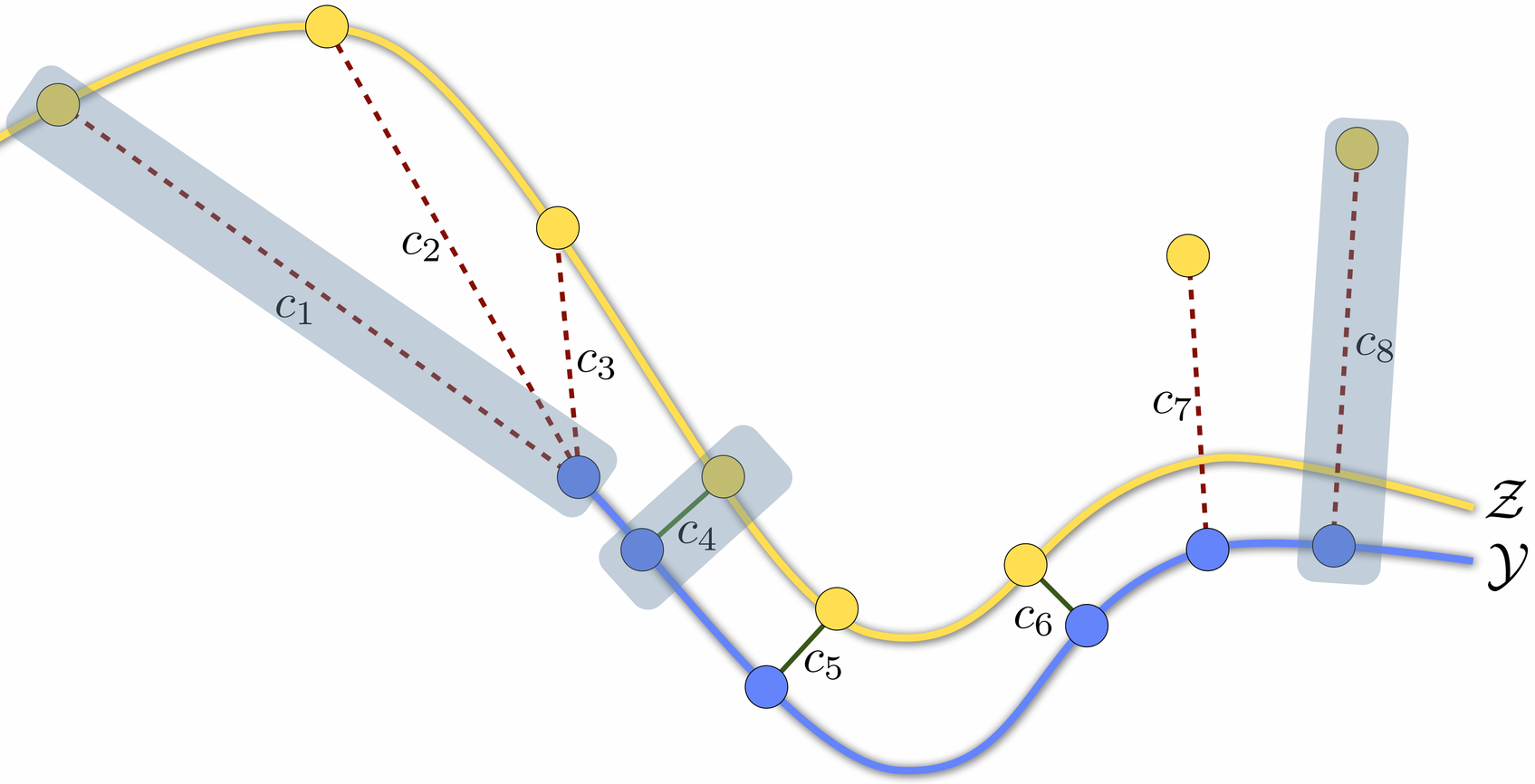
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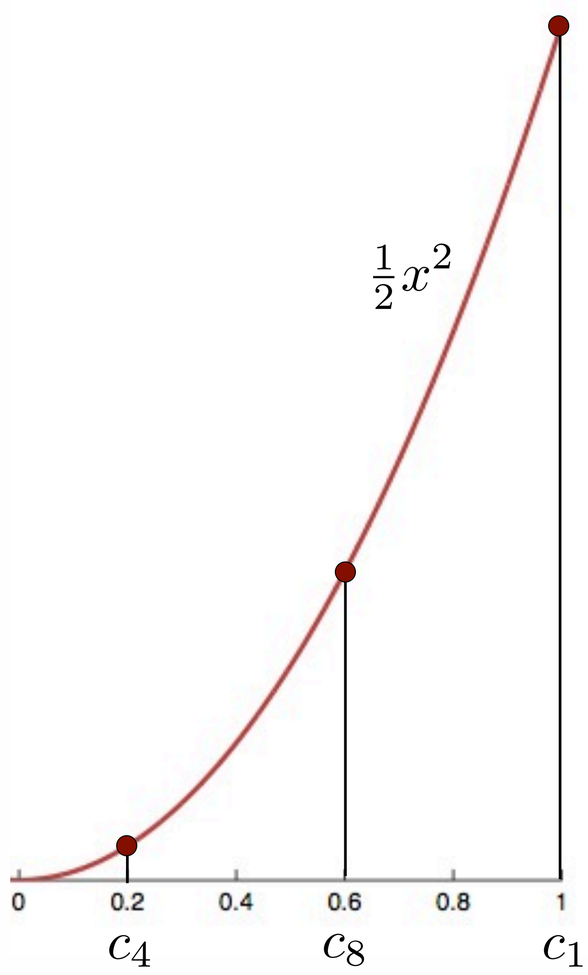




Robust Registration

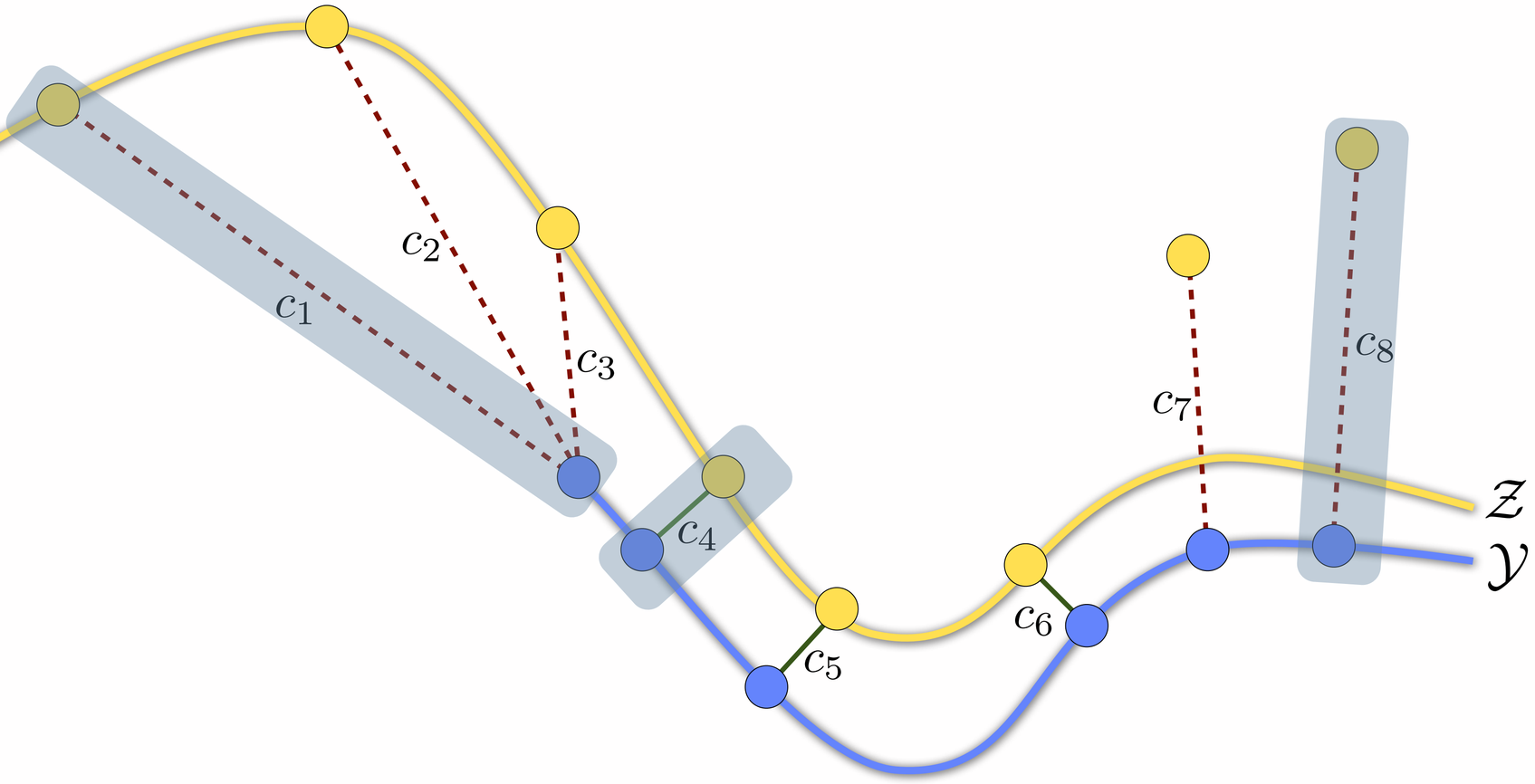
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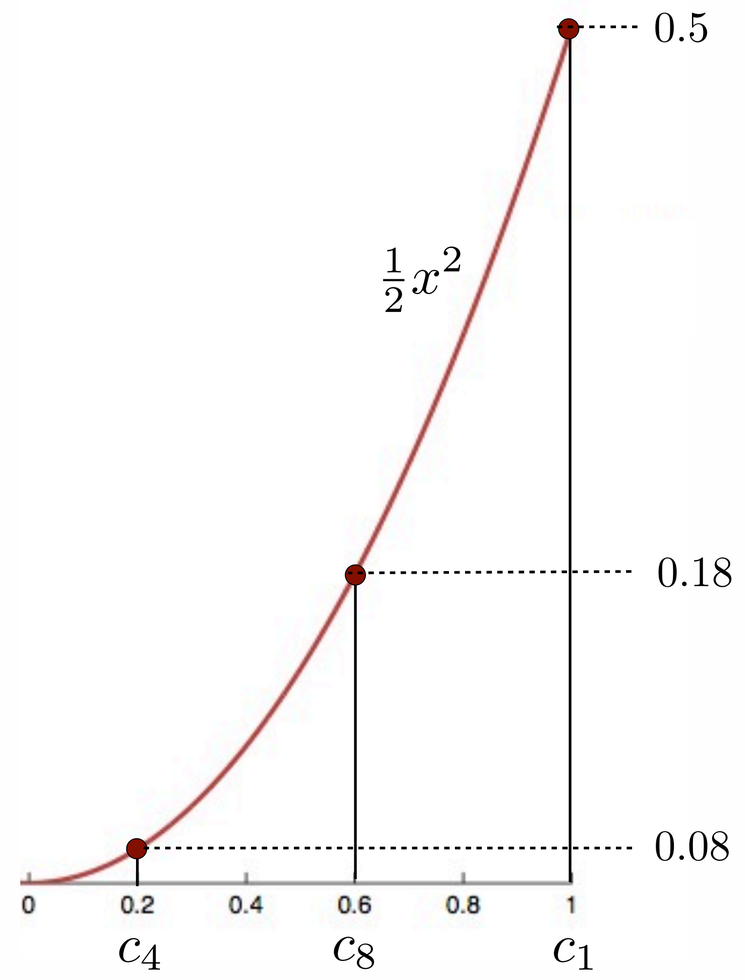




Robust Registration

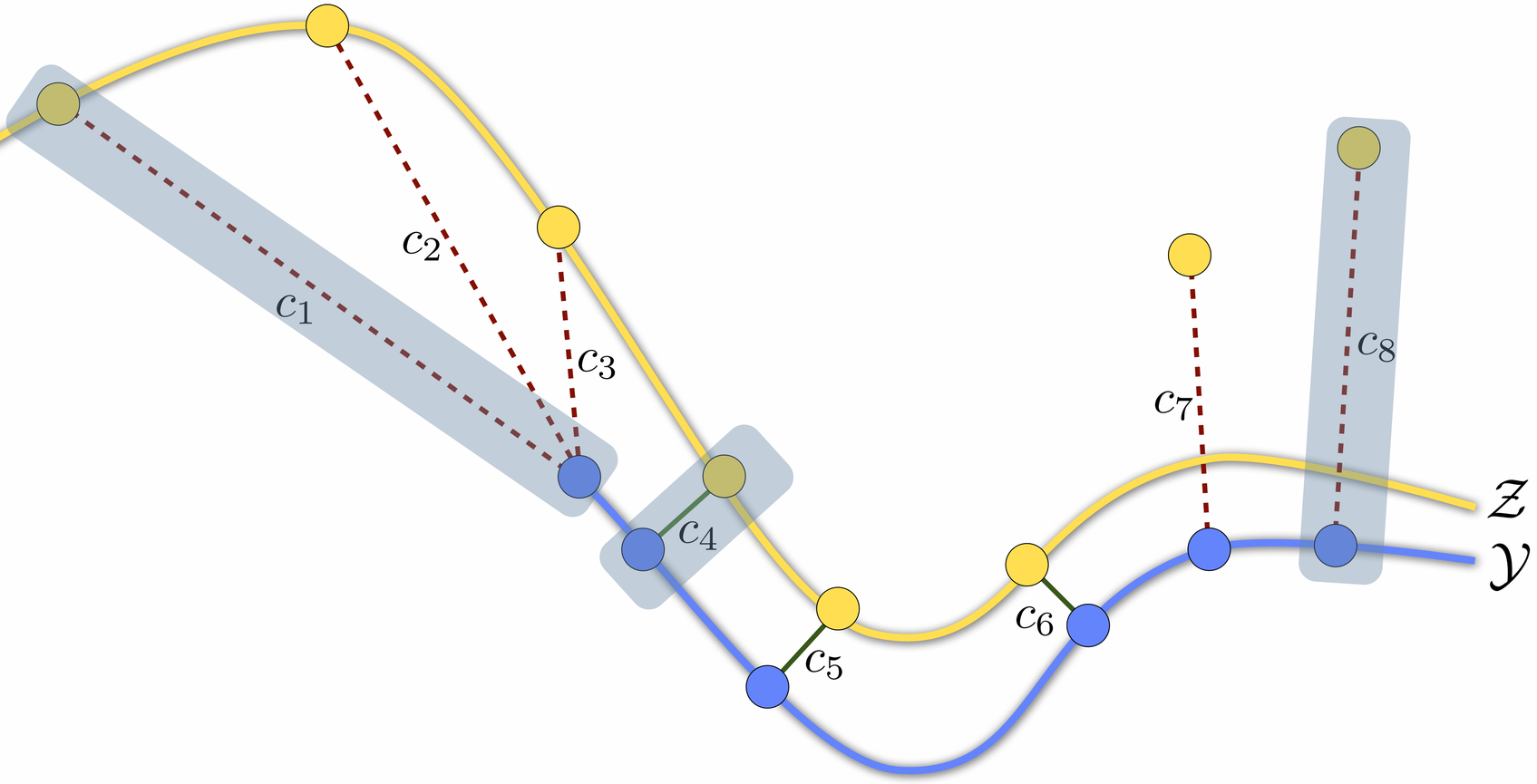
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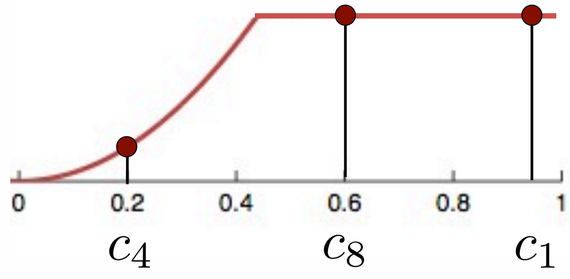




Robust Registration

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} c_i^2, \quad c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$





Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2$$

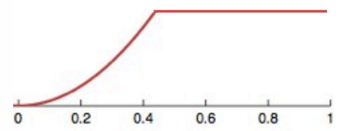
Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2 \quad \Longrightarrow \quad \arg\min_{Z} \sum_{i=1}^{n} \varphi(c_i)$$

Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2 \quad \Longrightarrow \quad \arg\min_{Z} \sum_{i=1}^{n} \varphi(c_i)$$

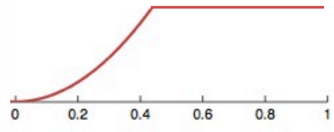
robust function



Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2 \quad \Longrightarrow \quad \arg\min_{Z} \sum_{i=1}^{n} \varphi(c_i)$$

robust function



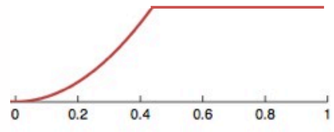
IRLS

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2 \quad \Longrightarrow \quad \arg\min_{Z} \sum_{i=1}^{n} \varphi(c_i)$$

robust function



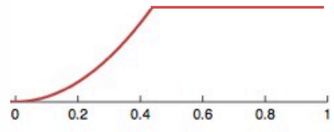
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Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2 \quad \Longrightarrow \quad \arg\min_{Z} \sum_{i=1}^{n} \varphi(c_i)$$

robust function



IRLS

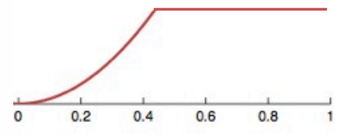
$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

weight function

Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2 \quad \Longrightarrow \quad \arg\min_{Z} \sum_{i=1}^{n} \varphi(c_i)$$

robust function



IRLS

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

weight function

previous iteration residuals

Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

$\varphi(x)$



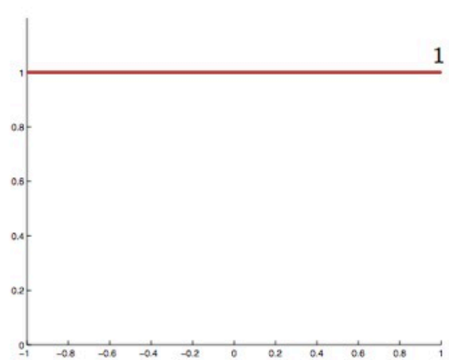
Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

$\varphi(x)$



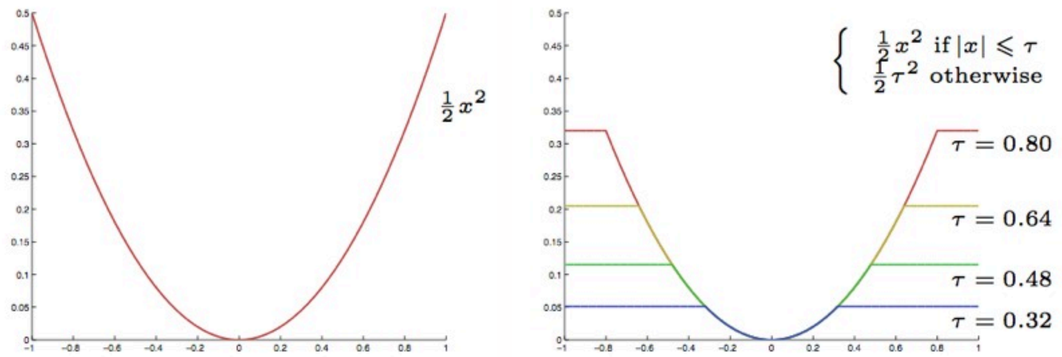
$w(x)$



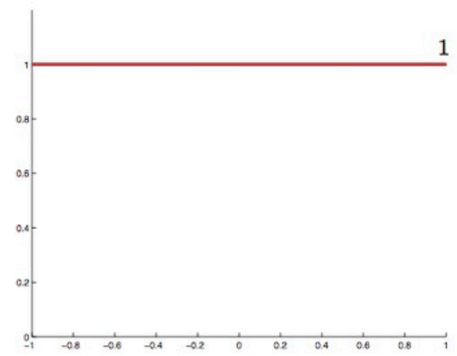
Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

$\varphi(x)$



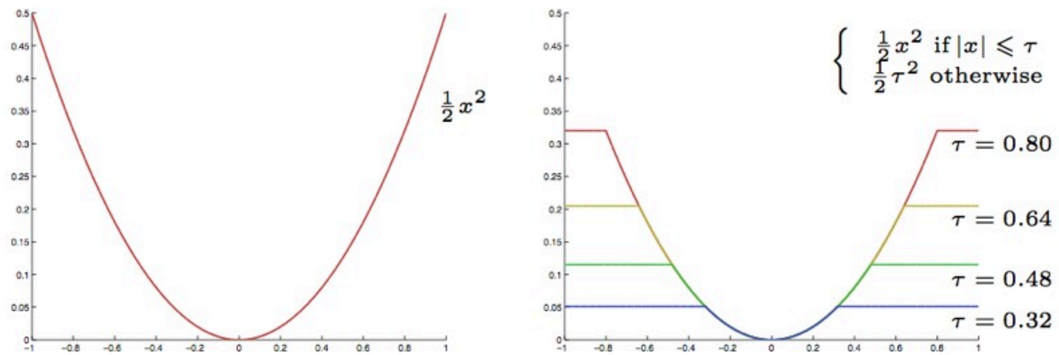
$w(x)$



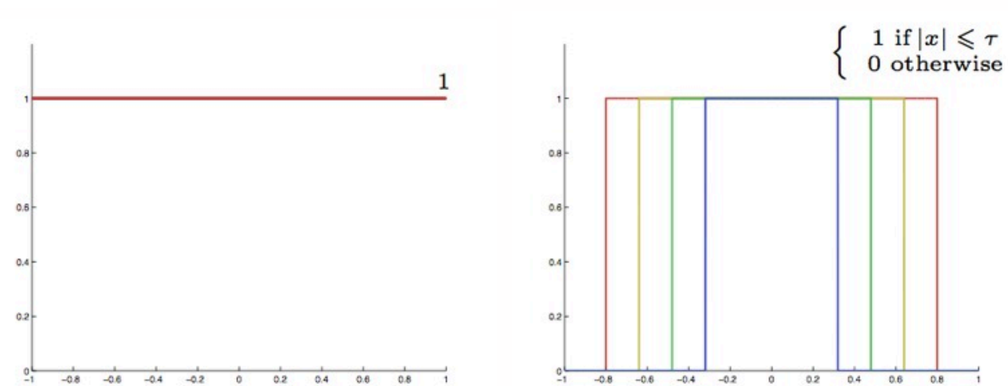
Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

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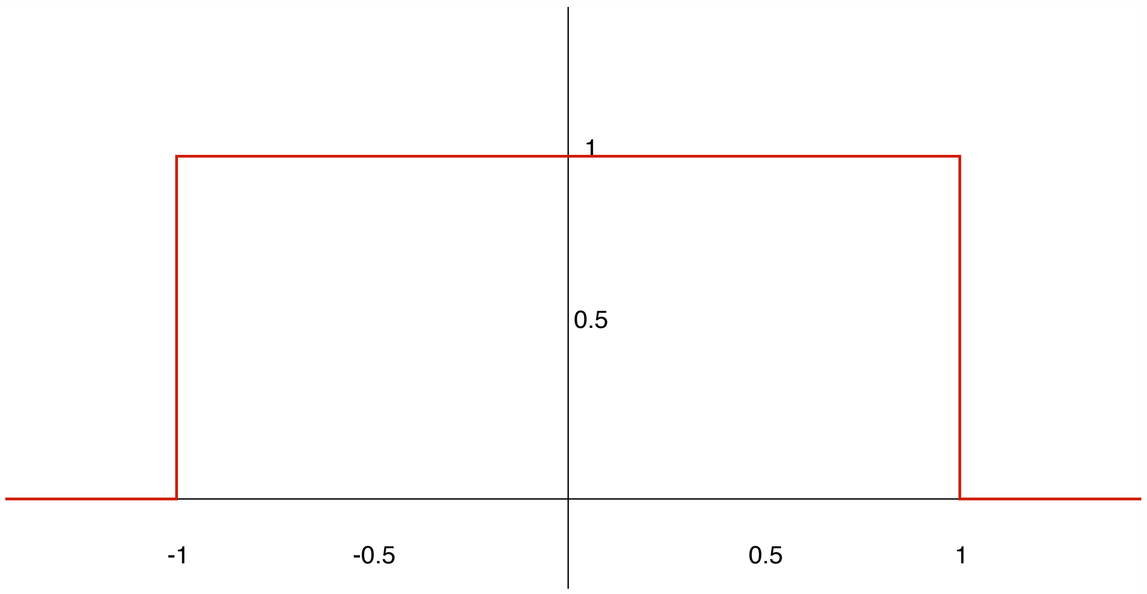


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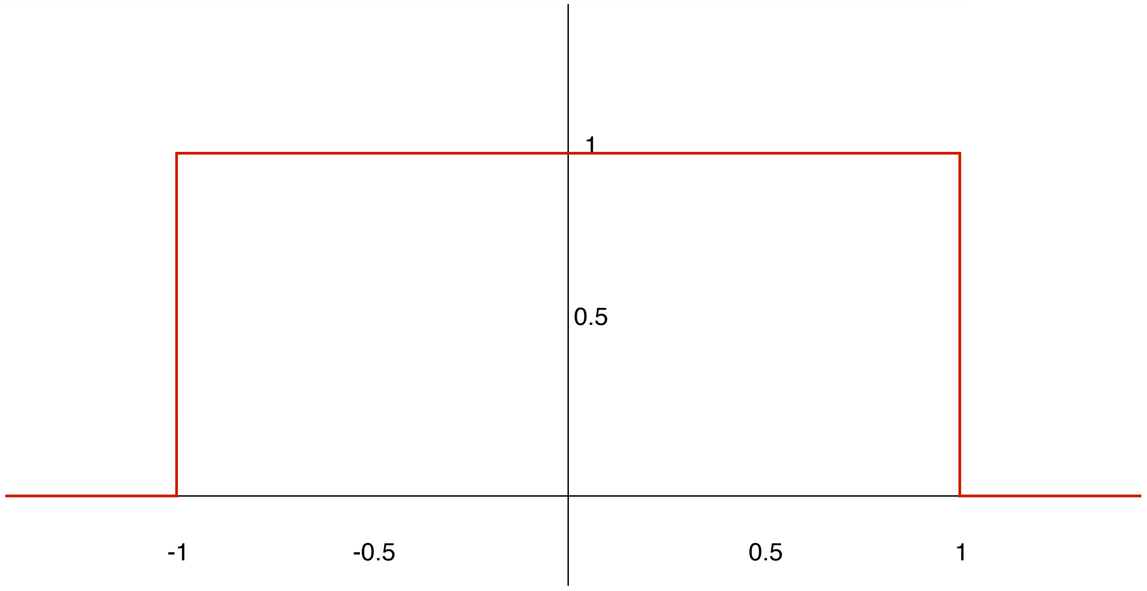
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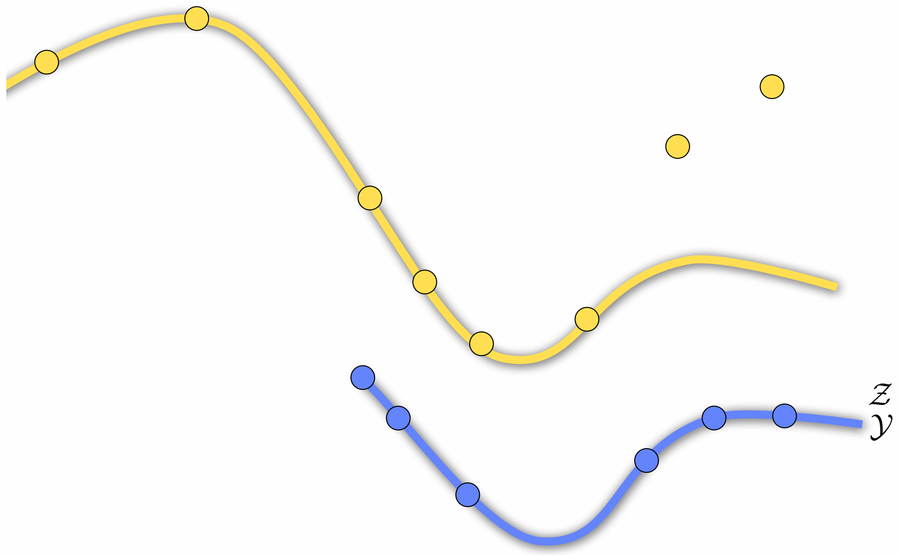
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Robust Registration

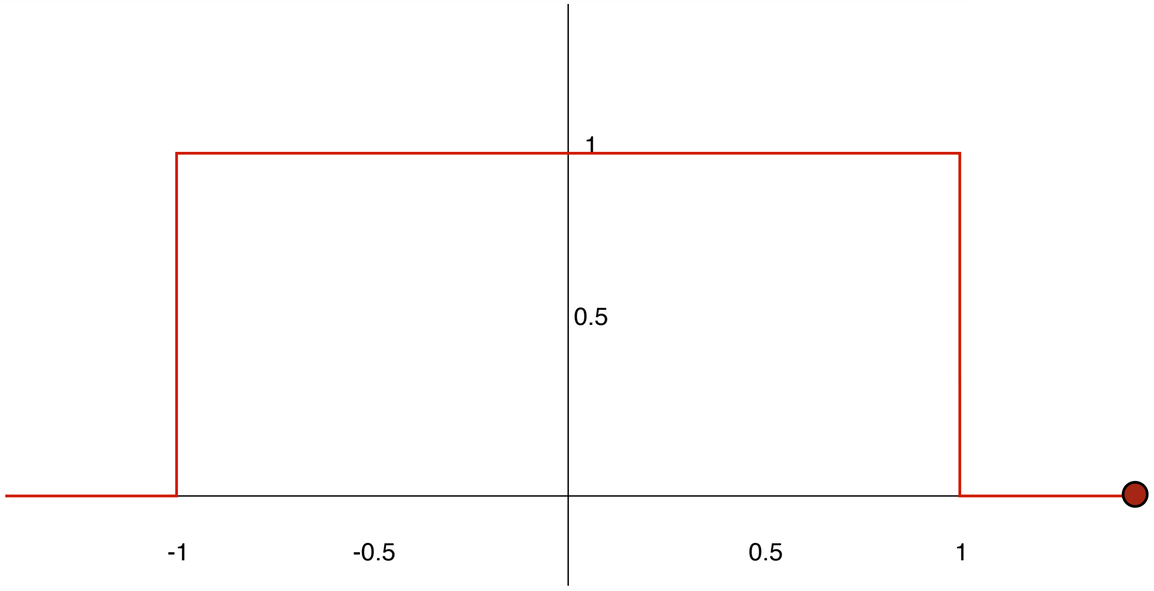
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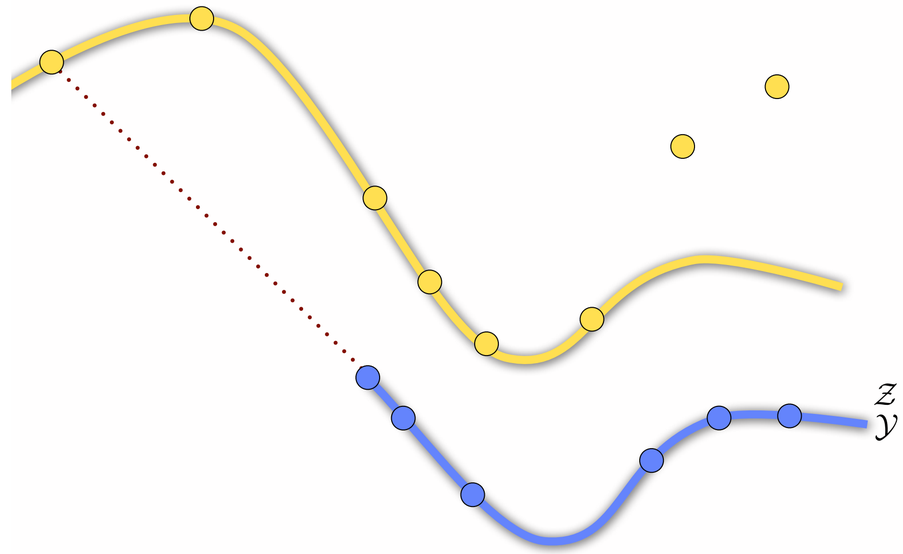




Robust Registration

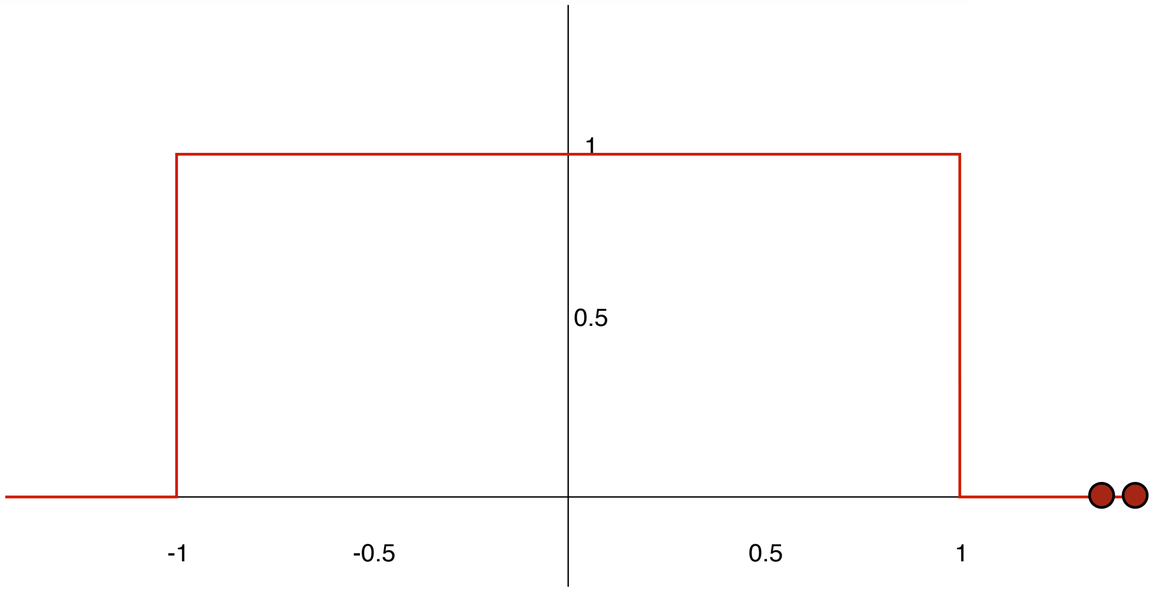
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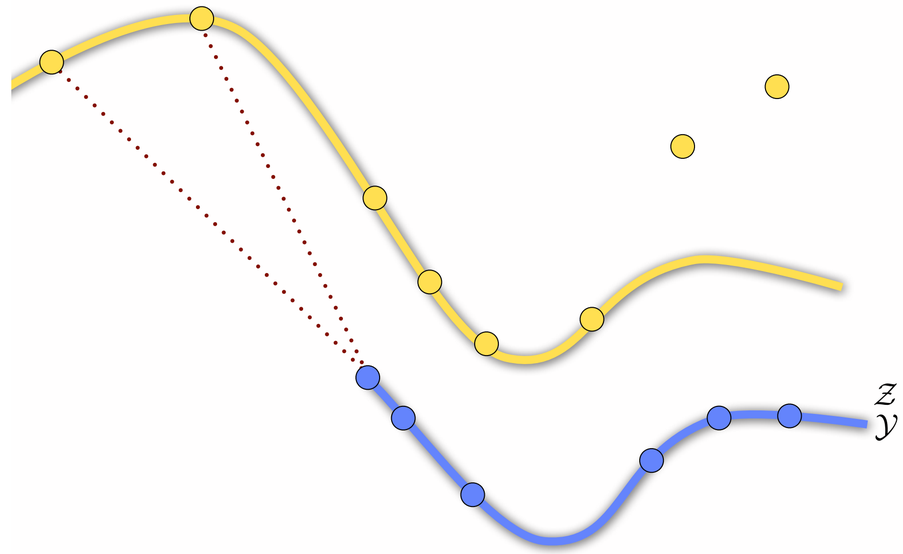




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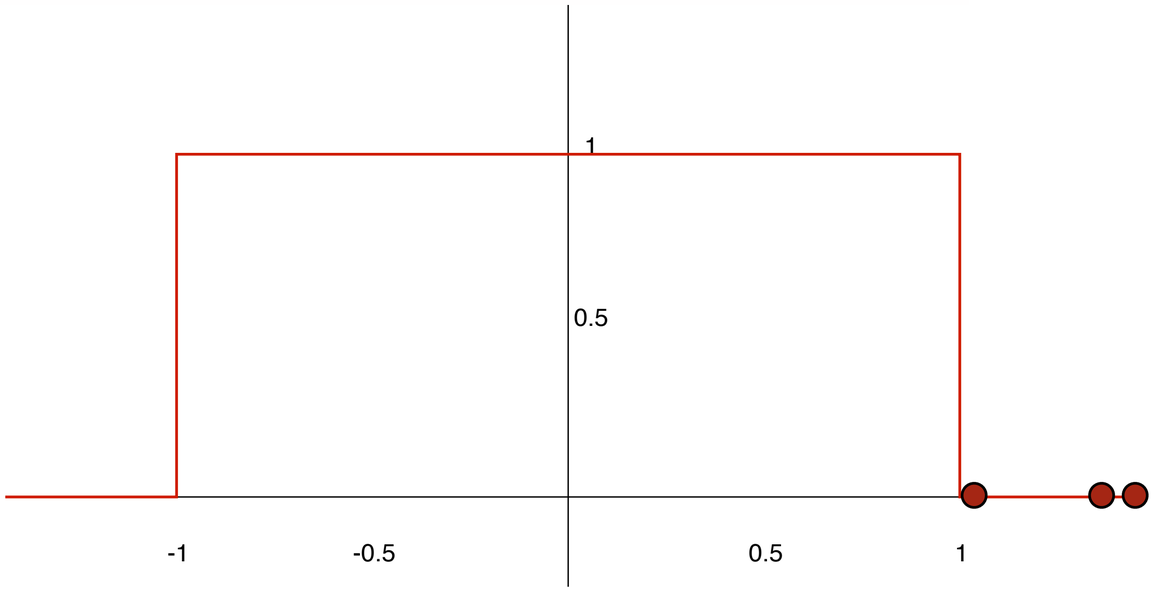
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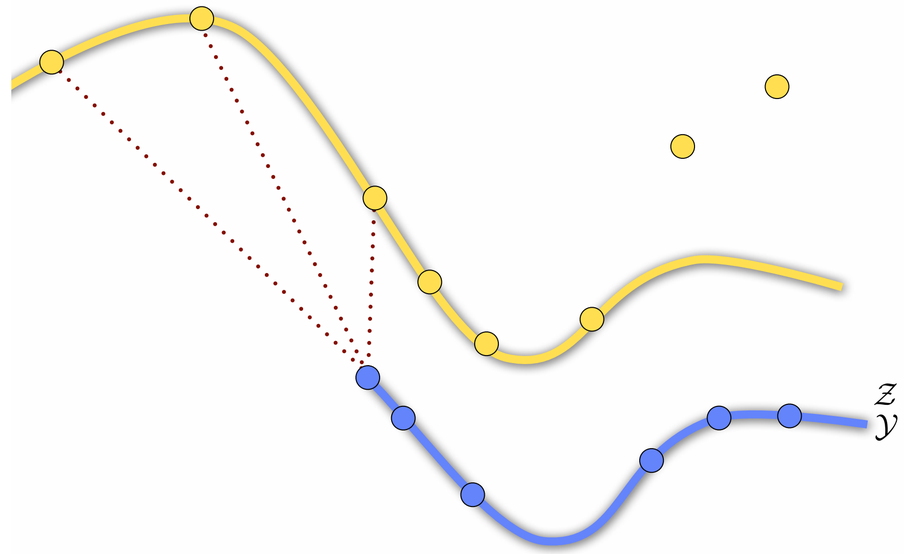




Robust Registration

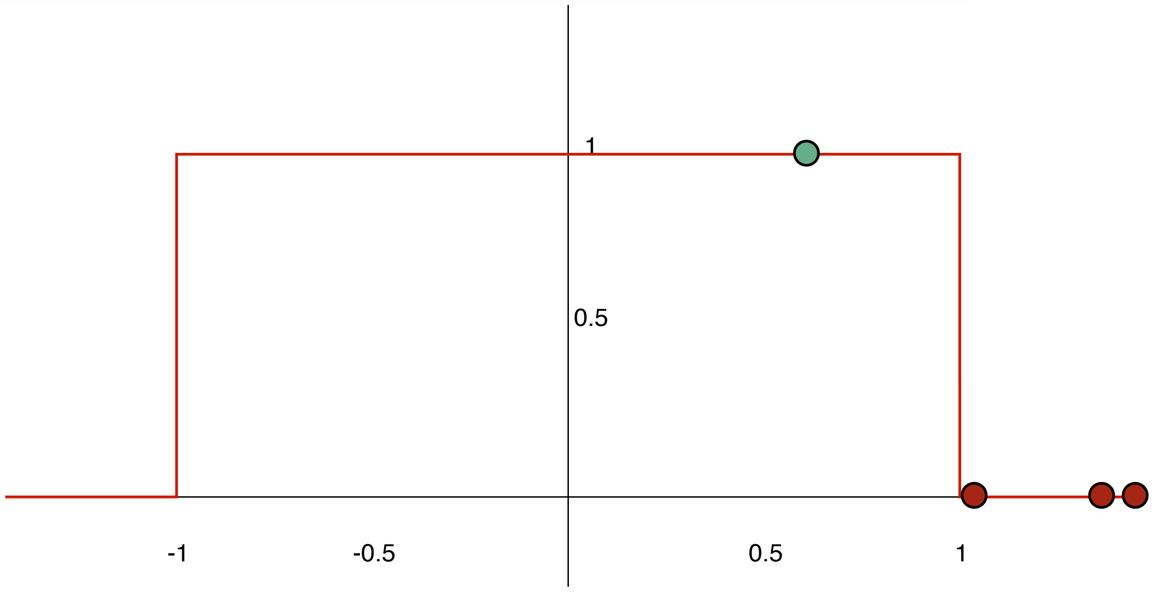
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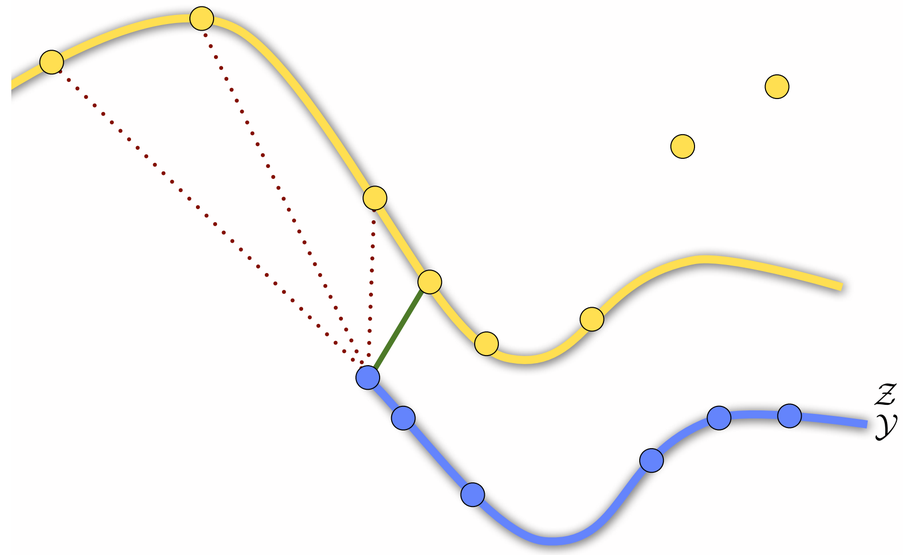




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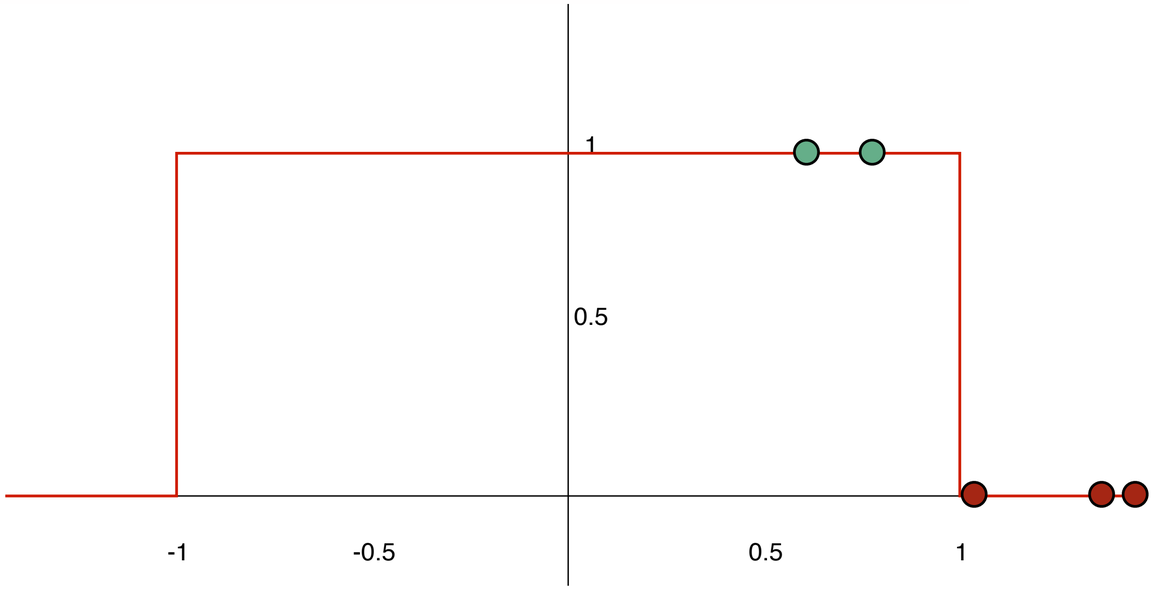
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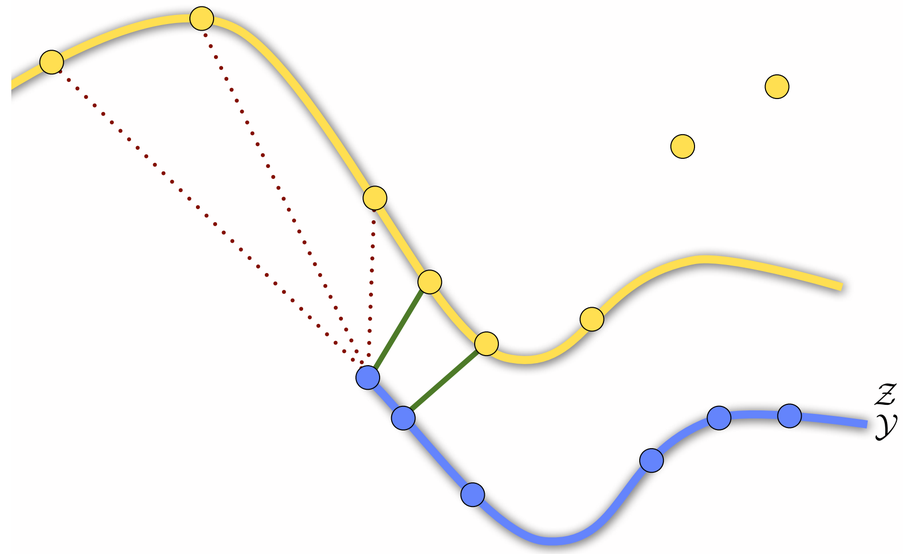




Robust Registration

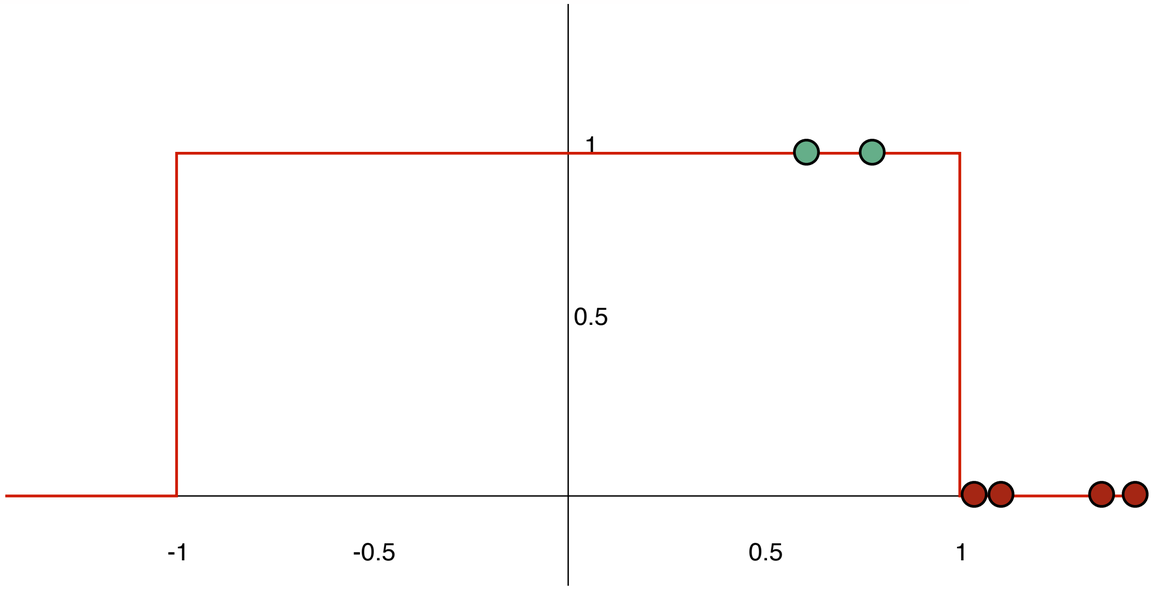
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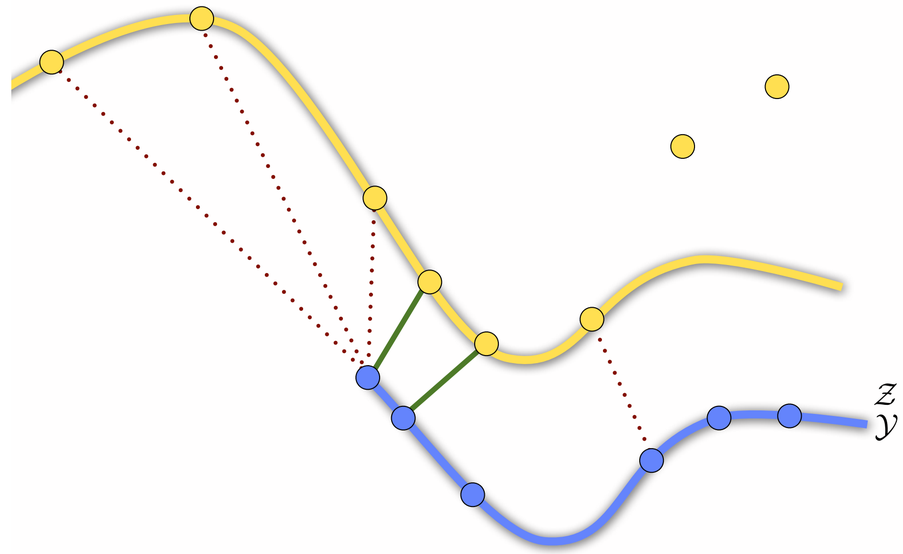




Robust Registration

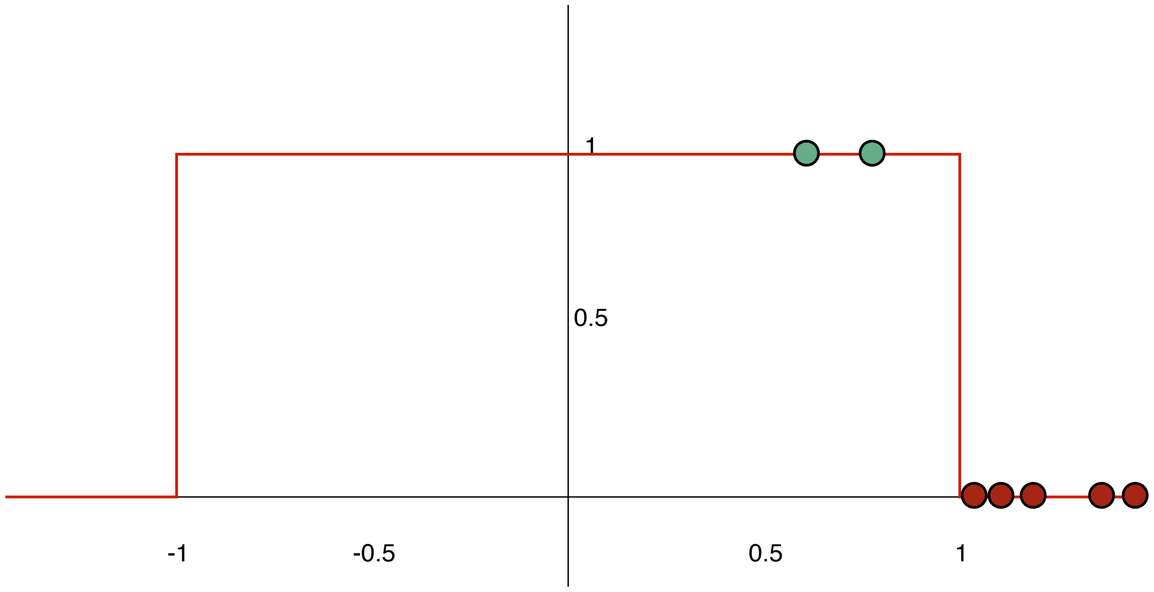
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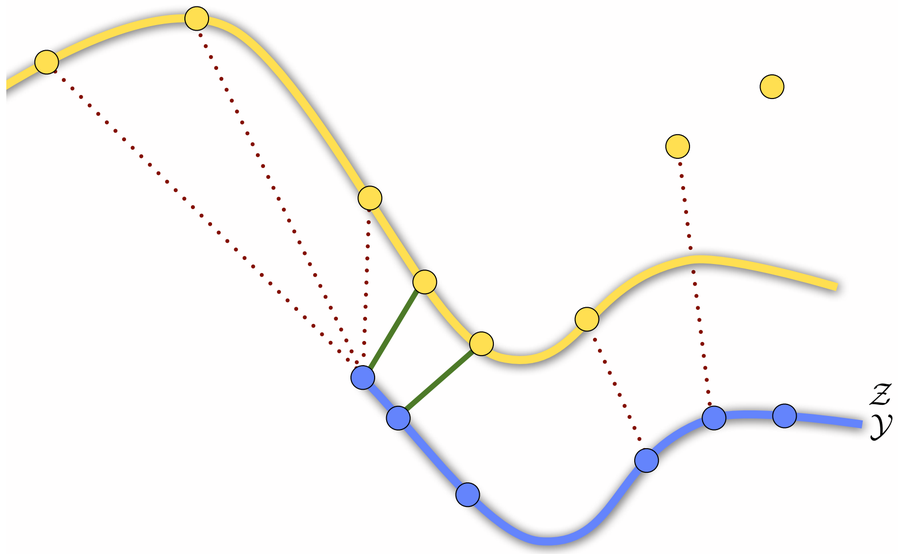




Robust Registration

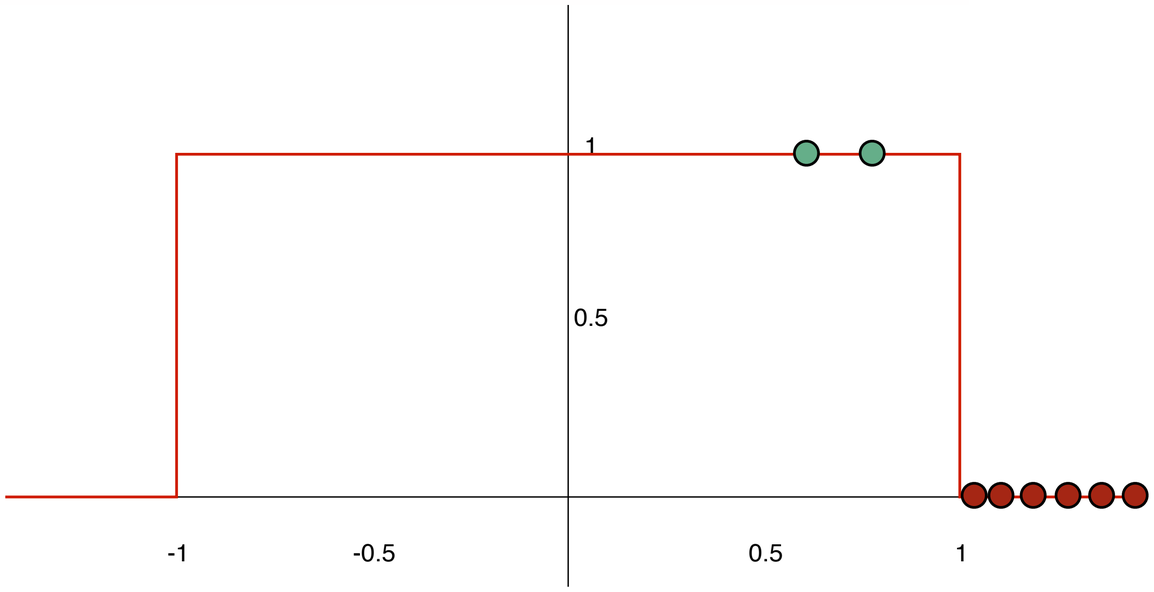
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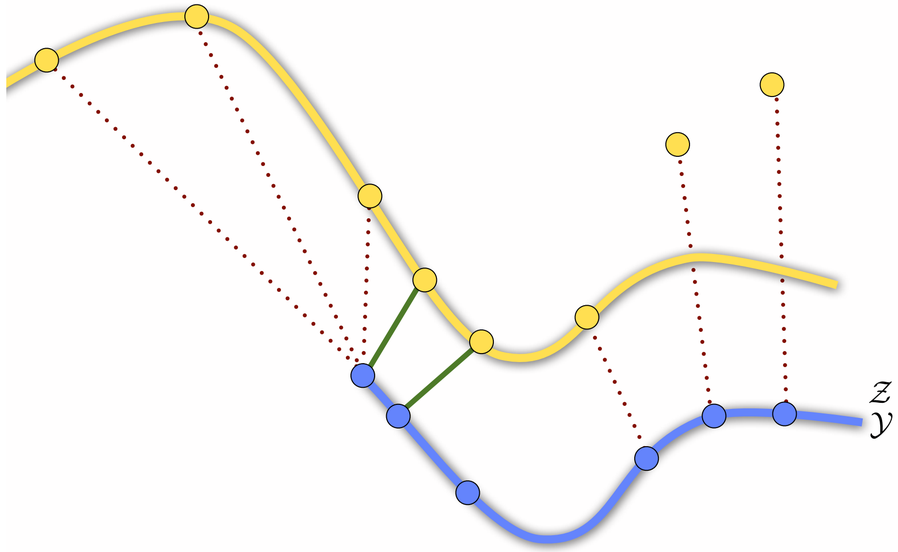




Robust Registration

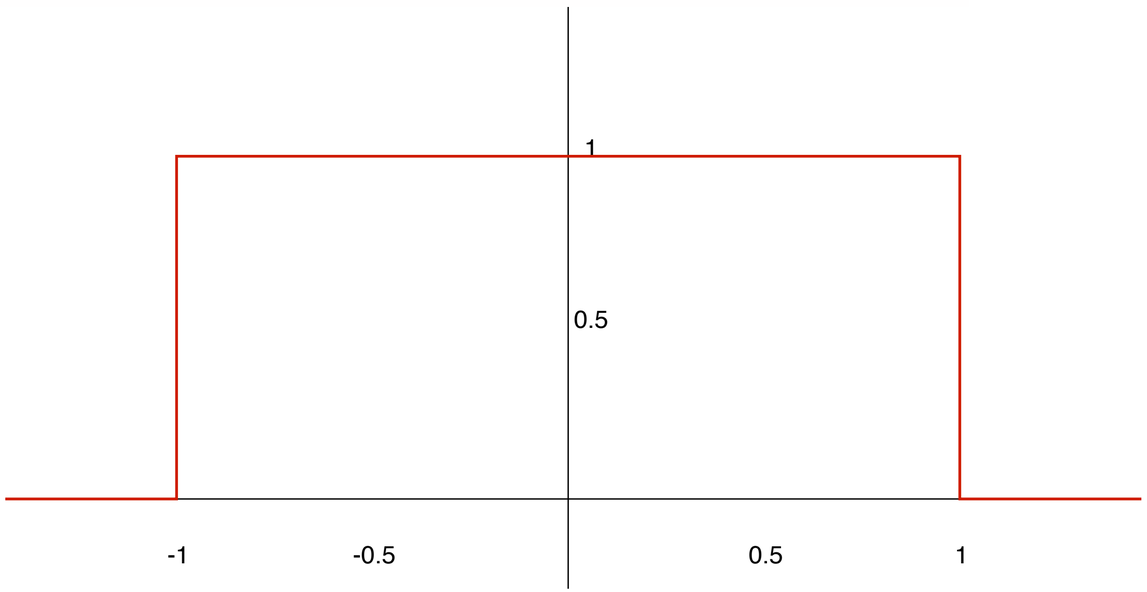
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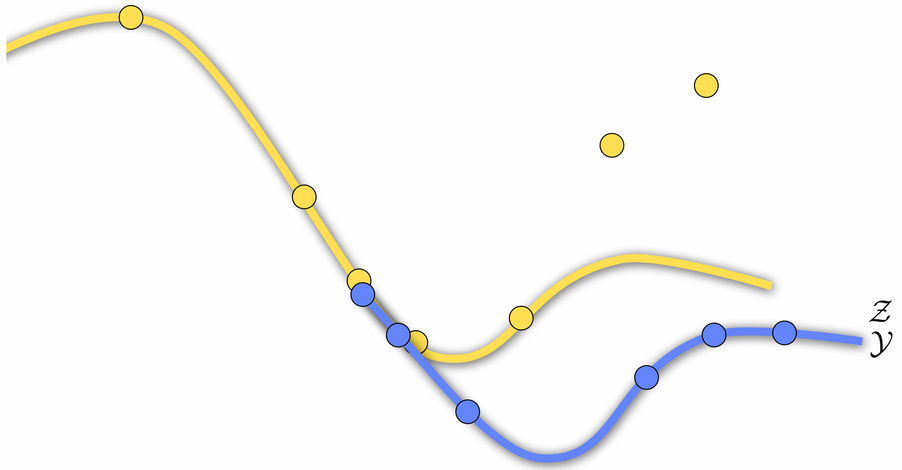




Robust Registration

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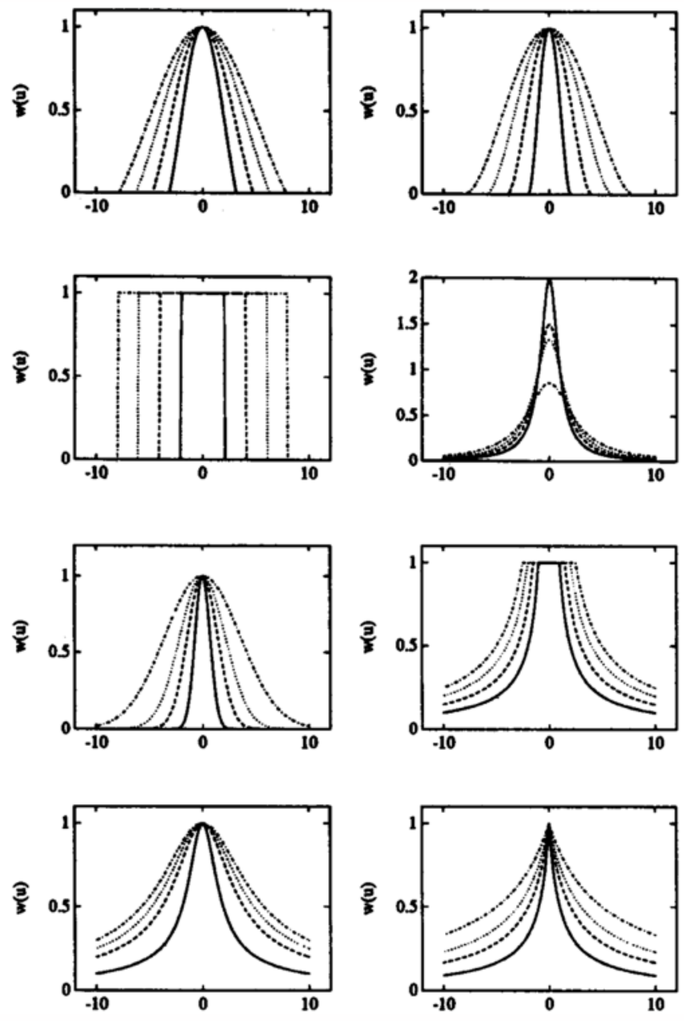




Robust Registration

TABLE I
WEIGHT FUNCTION

Estimator with Tuning Constant	Objective Function ρ	ψ-Function	Weight Function w	Range of u	Comments
Andrews' wave (a)	$a^2[1-\cos(u/a)]$ $2a^2$	$a\sin(u/a)$ 0	$(u/a)^{-1}\sin(u/a)$ 0	$\lvert u\rvert \le \pi a$ $\lvert u\rvert > \pi a$	$1.0 \le a \le 2.5$
Tukey's Biweight (b)	$(b^2/2)[1-(1-(u/b)^2)]^3$ $b^2/2$	$u[1-(u/b)^2]^2$ 0	$[1-(u/b)^2]^2$ 0	$\lvert u\rvert \le b$ $\lvert u\rvert > b$	$2.0 \le b \le 8.0$
Talwar (t)	$u^2/2$ $t^2/2$	u 0	1 0	$\lvert u\rvert \le t$ $\lvert u\rvert > t$	$2.0 \le t \le 8.0$
Student-t(f)			$(1+f)/(f+u^2)$		Derived in [1] by ML analysis
Cauchy(c)	$(c^2/2)\log[1+(u/c)^2]$	$u[1+(u/c)^2]^{-1}$	$[1+(u/c)^2]^{-1}$		special case of t-distribution
Welsch(w)	$(w^2/2)[1-\exp[-(u/w)^2]]$	$u\exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \le w \le 5.0$
Huber(h)	$u^2/2$ $h\lvert u\rvert - h^2/2$	u $h\ \mathrm{sig}(u)$	1 $h\lvert u\rvert^{-1}$	$\lvert u\rvert \le h$ $\lvert u\rvert > h$	$1.0 \le h \le 2.5$
Logistic(l)	$l^2\log[\cosh(u/l)]$	$l\tanh(u/l)$	$(u/l)^{-1}\tanh(u/l)$		$1.0 \le l \le 3.0$
Fair (f)	$f^2[\lvert u\rvert/f - \log(1+\lvert u\rvert/f)]$	$u(1+\lvert u\rvert/f)^{-1}$	$(1+\lvert u\rvert/f)^{-1}$		$1.0 \le f \le 5.0$

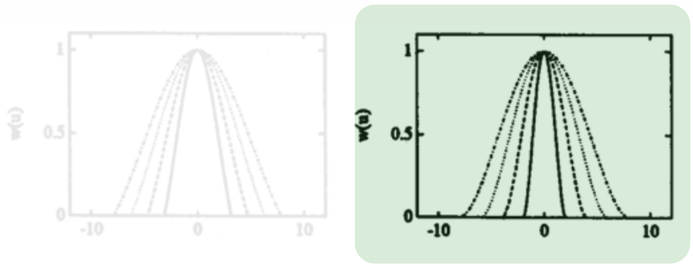


Mirza, Boyer: **Performance Evaluation of a Class of M-Estimators for Surface Parameter Estimation in Noisy Range Data,** *IEEE Transactions on Robotics and Automation*

Robust Registration

TABLE I
WEIGHT FUNCTION

Estimator with Tuning Constant	Objective Function ρ	ψ-Function	Weight Function w	Range of u	Comments
Andrews' wave (a)	$a^2[1-\cos(u/a)]$ $2a^2$	$a\sin(u/a)$ 0	$(u/a)^{-1}\sin(u/a)$ 0	$\lvert u\rvert \le \pi a$ $\lvert u\rvert > \pi a$	$1.0 \le a \le 2.5$
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Talwar (t)	$u^2/2$ $t^2/2$	u 0	1 0	$\lvert u\rvert \le t$ $\lvert u\rvert > t$	$2.0 \le t \le 8.0$
Student-t(f)			$(1+f)/(f$		special case of t-distribution
Cauchy(c)	$(c^2/2)\log[1+(u/c)^2]$	$u[1+(u/c)^2]^{-1}$	$[1+(u/c)^2]^{-1}$		
Welsch(w)	$(w^2/2)[1-\exp[-(u/w)^2]]$	$u\exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \le w \le 5.0$
Huber(h)	$u^2/2$ $h\lvert u\rvert - h^2/2$	u $h\ \mathrm{sig}(u)$	1 $h\lvert u\rvert^{-1}$	$\lvert u\rvert \le h$ $\lvert u\rvert > h$	$1.0 \le h \le 2.5$
Logistic(l)	$l^2\log[\cosh(u/l)]$	$l\tanh(u/l)$	$(u/l)^{-1}\tanh(u/l)$		$1.0 \le l \le 3.0$
Fair (f)	$f^2[\lvert u\rvert/f - \log(1+\lvert u\rvert/f)]$	$u(1+\lvert u\rvert/f)^{-1}$	$(1+\lvert u\rvert/f)^{-1}$		$1.0 \le f \le 5.0$



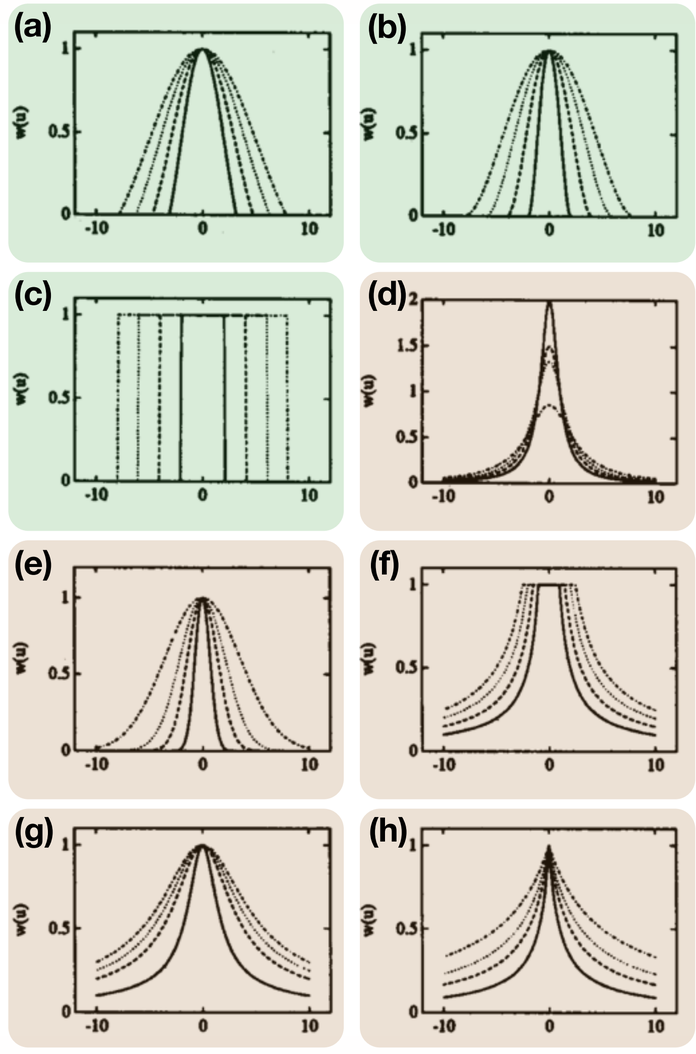
DEMO

Mirza, Boyer: **Performance Evaluation of a Class of M-Estimators for Surface Parameter Estimation in Noisy Range Data,** *IEEE Transactions on Robotics and Automation 1993*

Robust Registration

TABLE I
WEIGHT FUNCTION

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Andrews' wave (a)	$a^2[1-\cos(u/a)]$ $2a^2$	$a\sin(u/a)$ 0	$(u/a)^{-1}\sin(u/a)$ 0	$\lvert u\rvert \le \pi a$ $\lvert u\rvert > \pi a$	$1.0 \le a \le 2.5$ (b)
Tukey's Biweight (b)	$(b^2/2)[1-(1-(u/b)^2)]^3$ $b^2/2$	$u[1-(u/b)^2]^2$ 0	$[1-(u/b)^2]^2$ 0	$\lvert u\rvert \le b$ $\lvert u\rvert > b$	$2.0 \le b \le 8.0$ (c)
Talwar (t)	$u^2/2$ $t^2/2$	u 0	1 0	$\lvert u\rvert \le t$ $\lvert u\rvert > t$	$2.0 \le t \le 8.0$
Student-t(f)			$(1+f)/(f+u^2)$		Derived in [1] by ML analysis (d)
Cauchy(c)	$(c^2/2)\log[1+(u/c)^2]$	$u[1+(u/c)^2]^{-1}$	$[1+(u/c)^2]^{-1}$		special case of t-distribution
Welsch(w)	$(w^2/2)[1-\exp[-(u/w)^2]]$	$u\exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \le w \le 5.0$ (e)
Huber(h)	$u^2/2$ $h\lvert u\rvert - h^2/2$	u $h\ \mathrm{sig}(u)$	1 $h\lvert u\rvert^{-1}$	$\lvert u\rvert \le h$ $\lvert u\rvert > h$	$1.0 \le h \le 2.5$ (f)
Logistic(l)	$l^2\log[\cosh(u/l)]$	$l\tanh(u/l)$	$(u/l)^{-1}\tanh(u/l)$		$1.0 \le l \le 3.0$ (g)
Fair (f)	$f^2[\lvert u\rvert/f - \log(1+\lvert u\rvert/f)]$	$u(1+\lvert u\rvert/f)^{-1}$	$(1+\lvert u\rvert/f)^{-1}$		$1.0 \le f \le 5.0$ (h)



Local

Global

Mirza, Boyer: **Performance Evaluation of a Class of M-Estimators for Surface Parameter Estimation in Noisy Range Data,** *IEEE Transactions on Robotics and Automation 1993*

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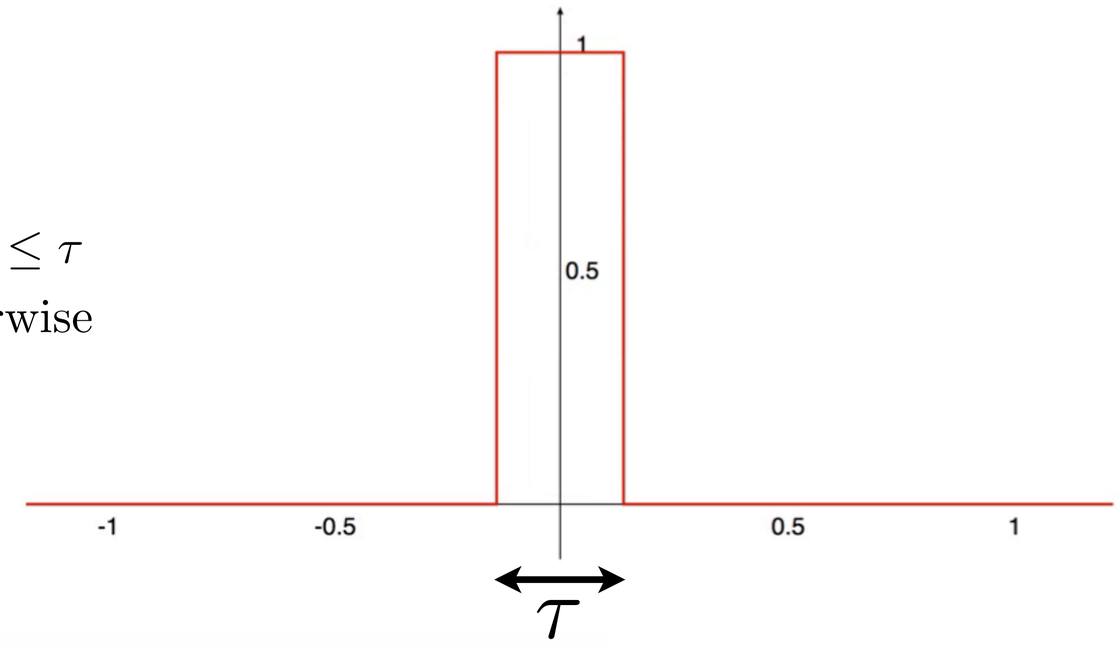
- Local Support

$$w(x) = \begin{cases} 1 & \text{if } |x| \leq \tau \\ 0 & \text{otherwise} \end{cases}$$

Robust Registration

- Local Support

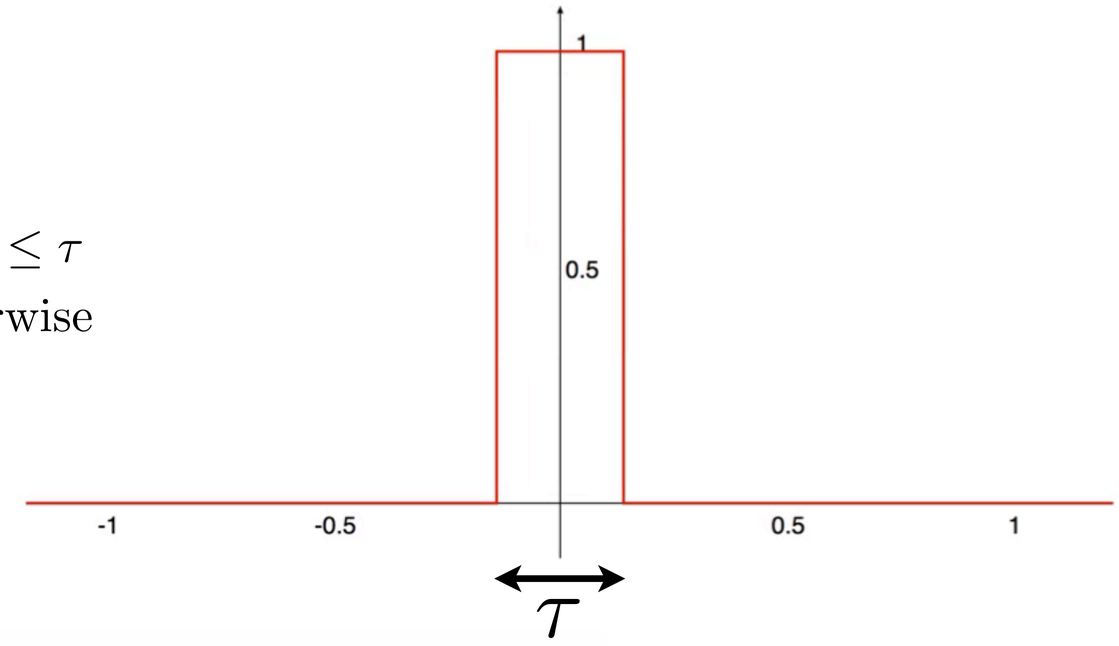
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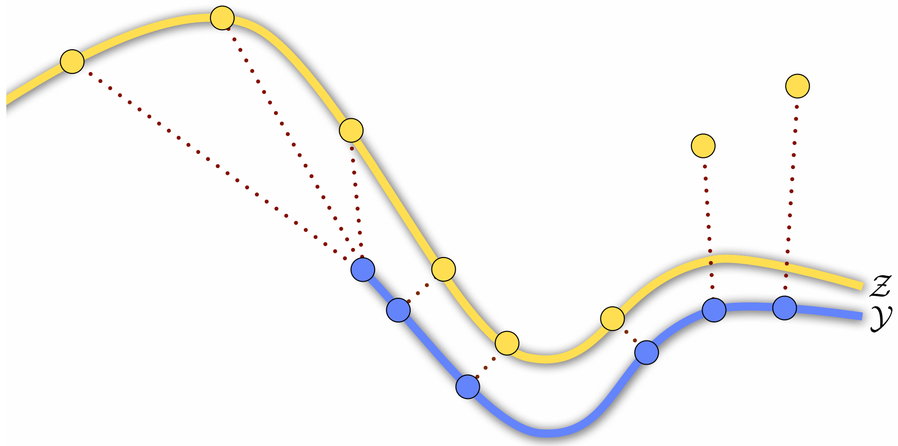


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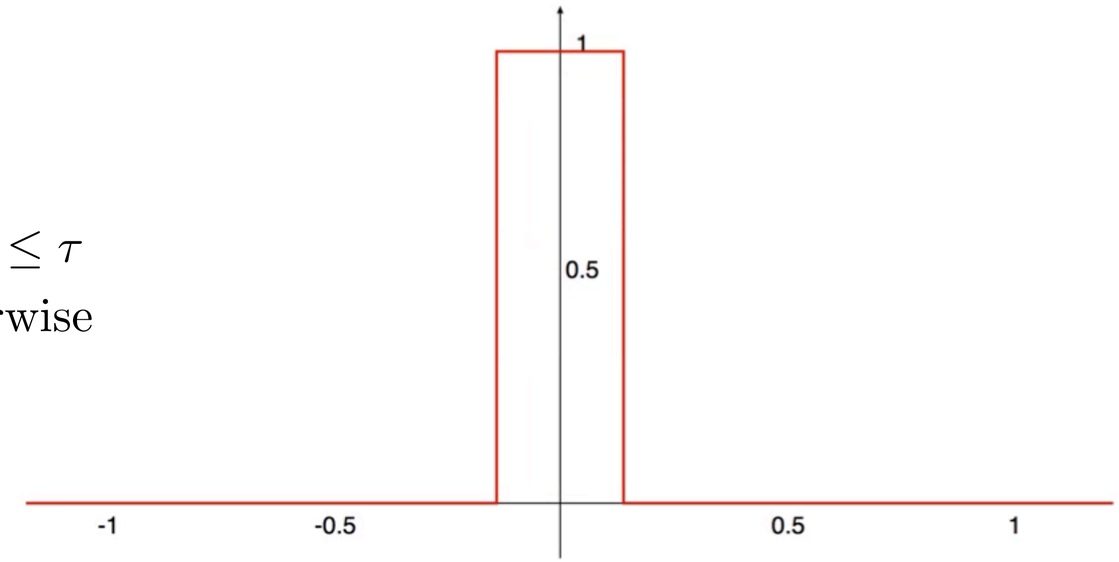


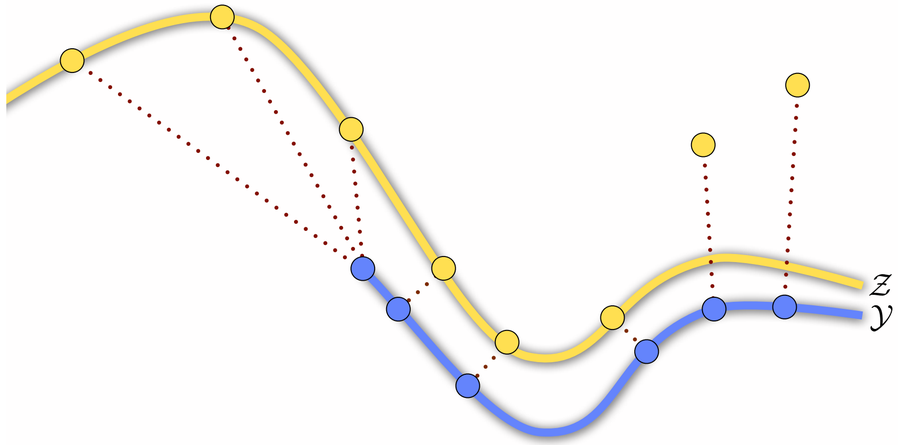


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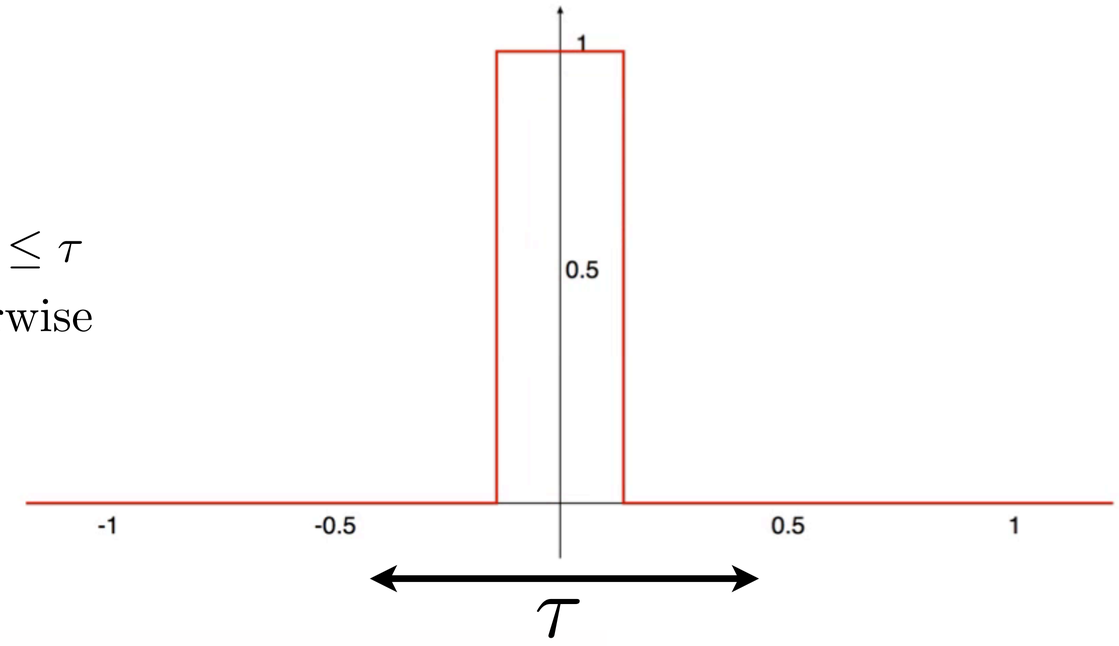


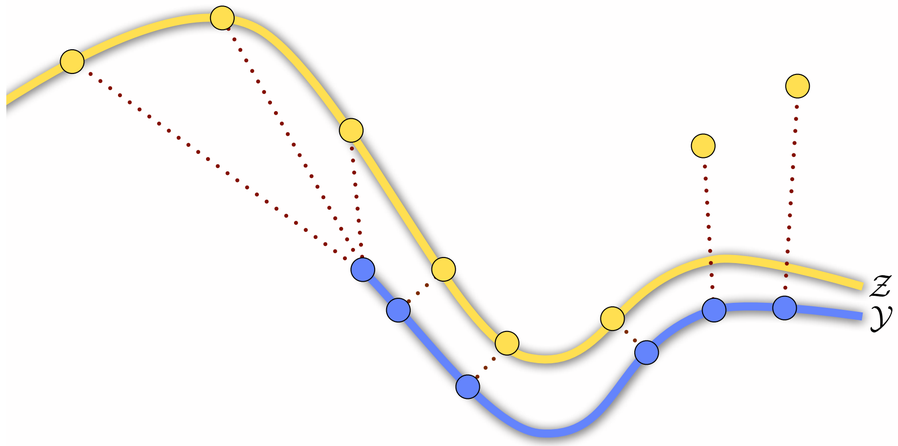


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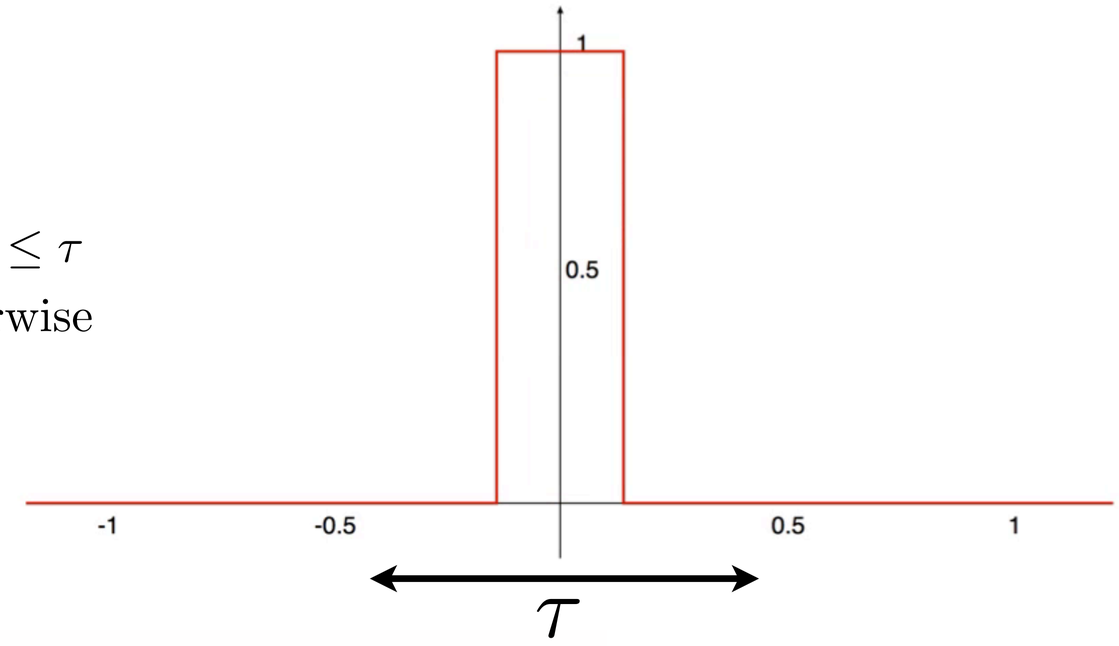


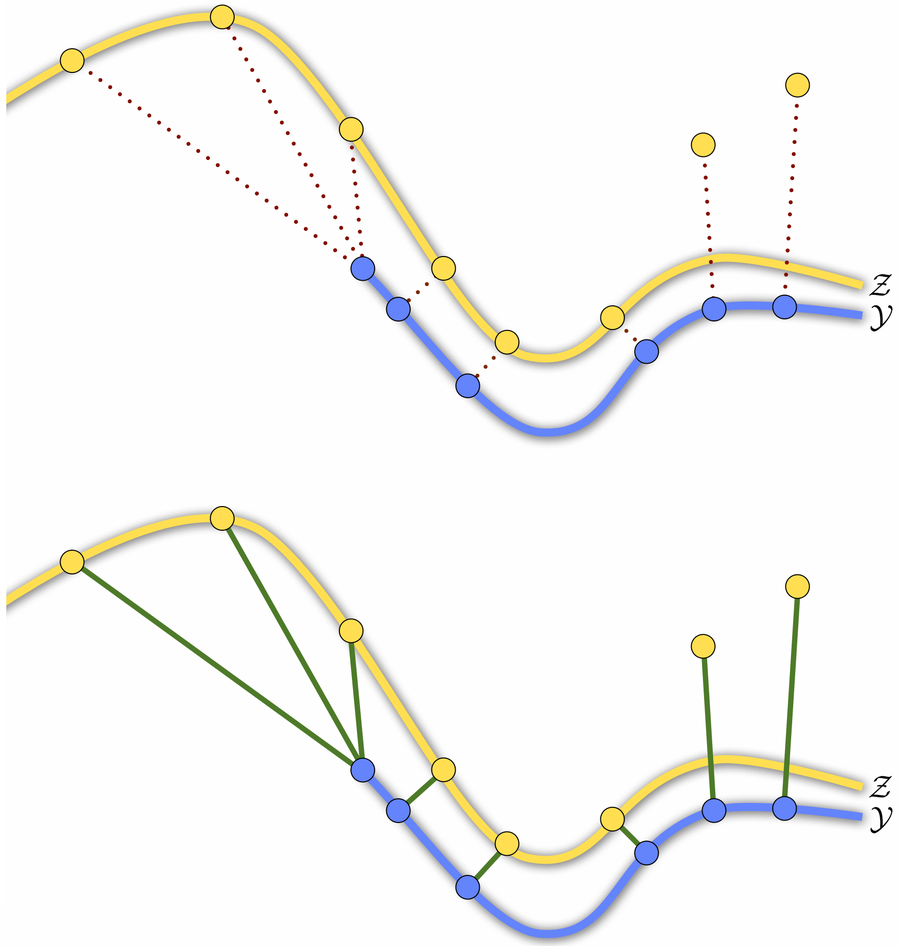


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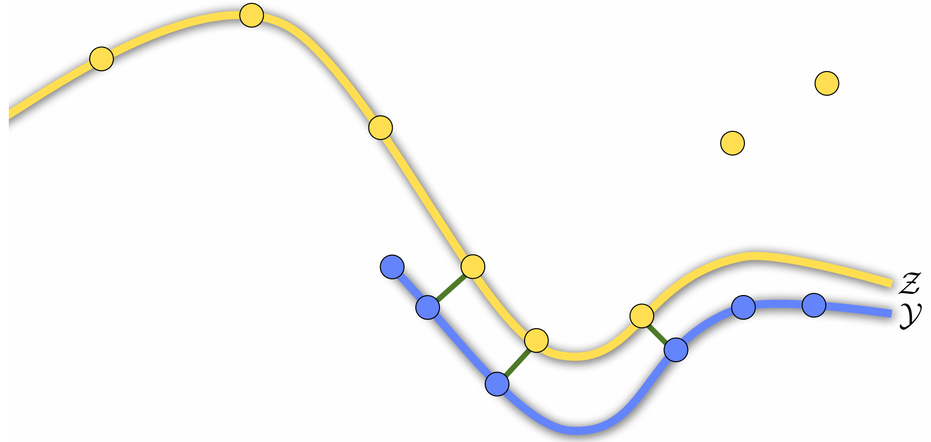


Robust Registration

- Trimmed Metrics
  - *Known number of inlier N*

N=3

$$w(x) = \begin{cases} 1 & \text{if } |x| \leq \tau \\ 0 & \text{otherwise} \end{cases}$$



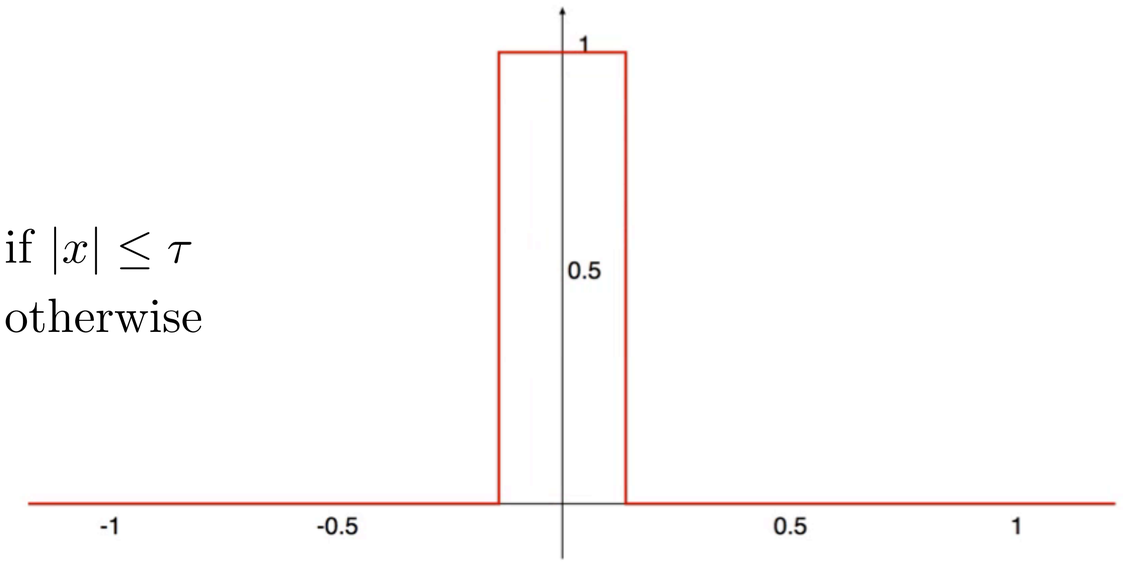
Chetverikov, Svirko, Stepanov, Krsek
The Trimmed Iterative Closest Point Algorithm,
International Conference on Pattern Recognition 2002

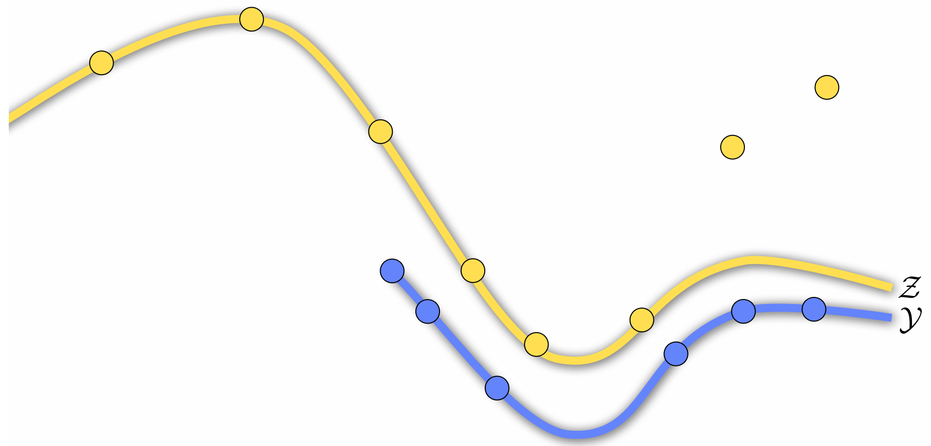
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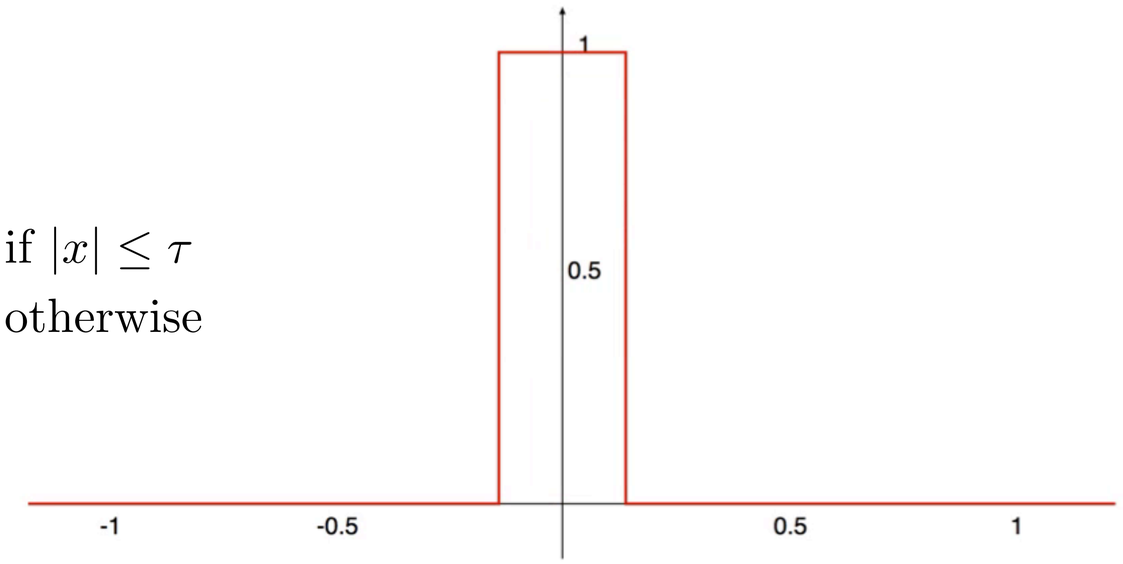


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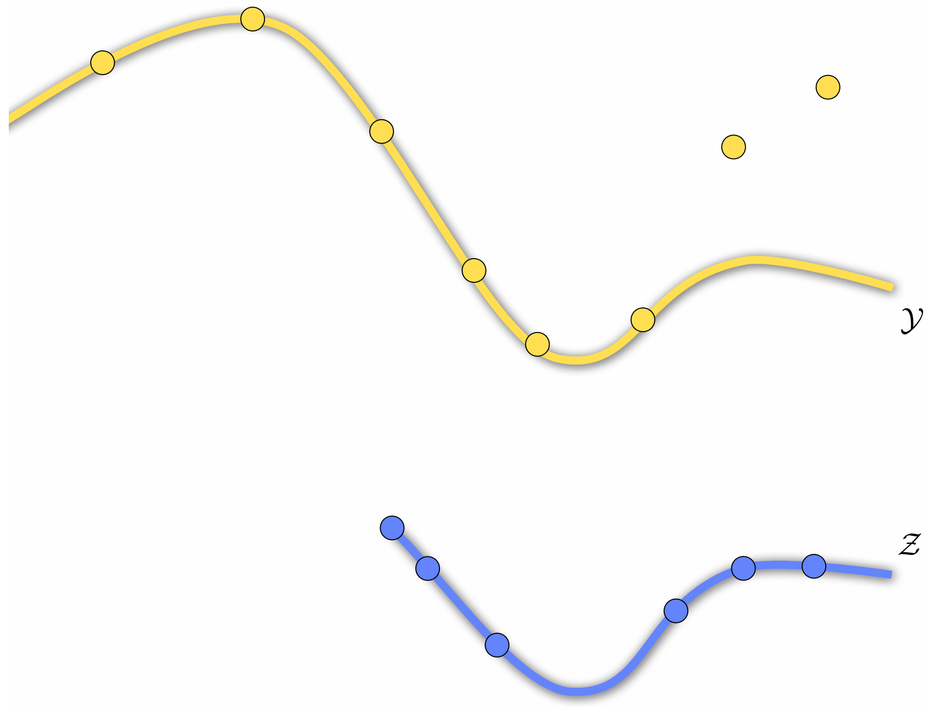
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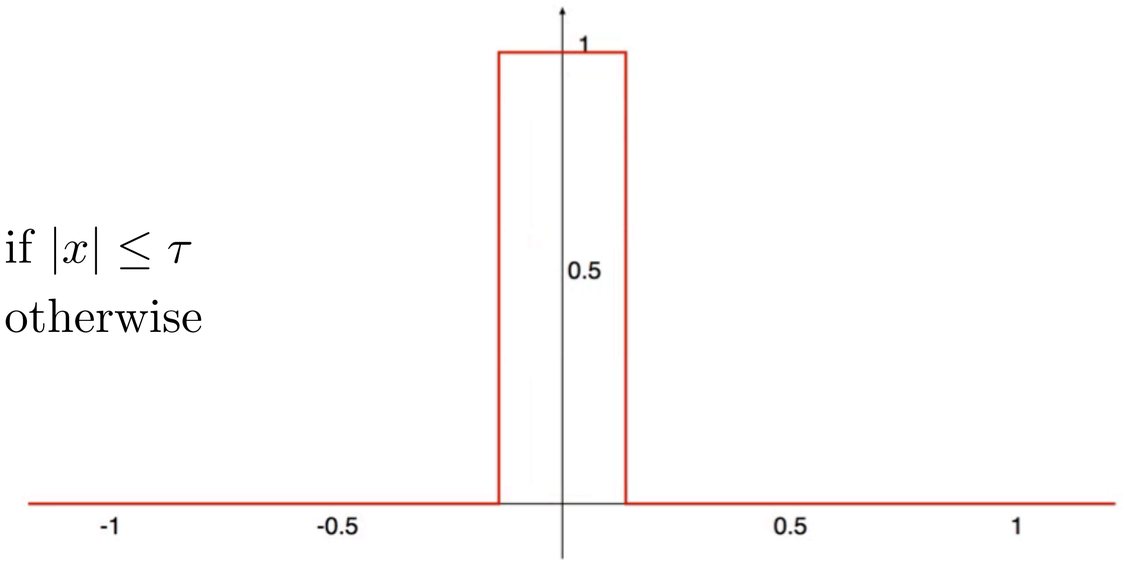


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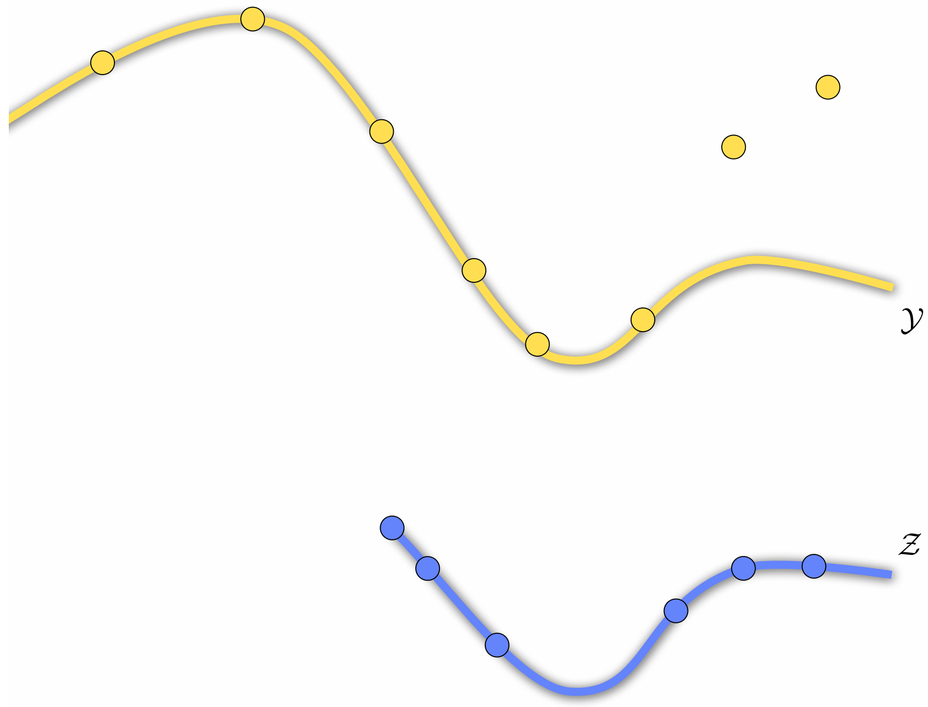
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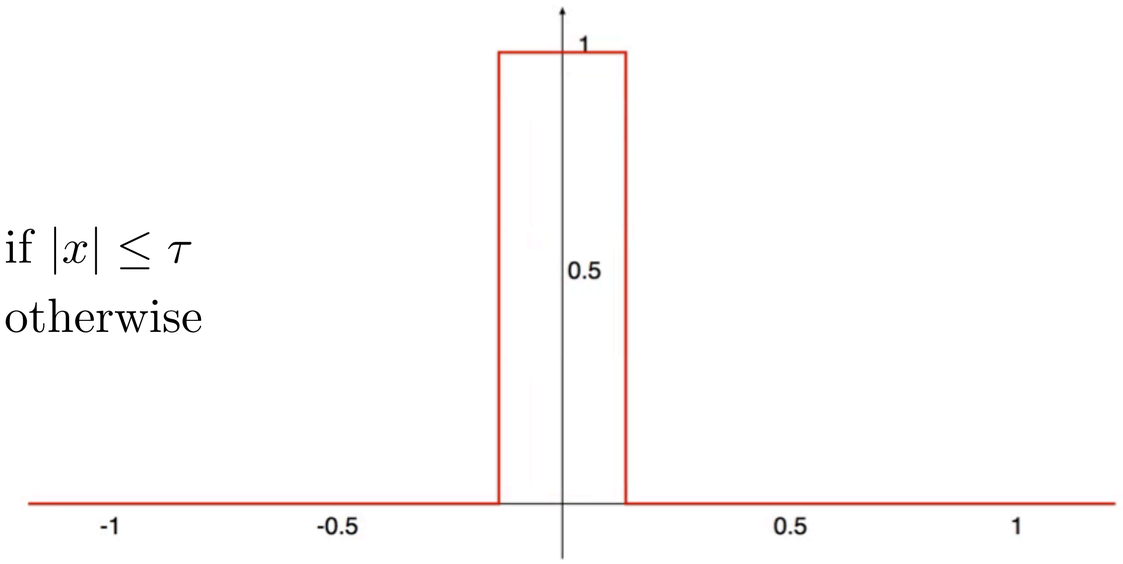


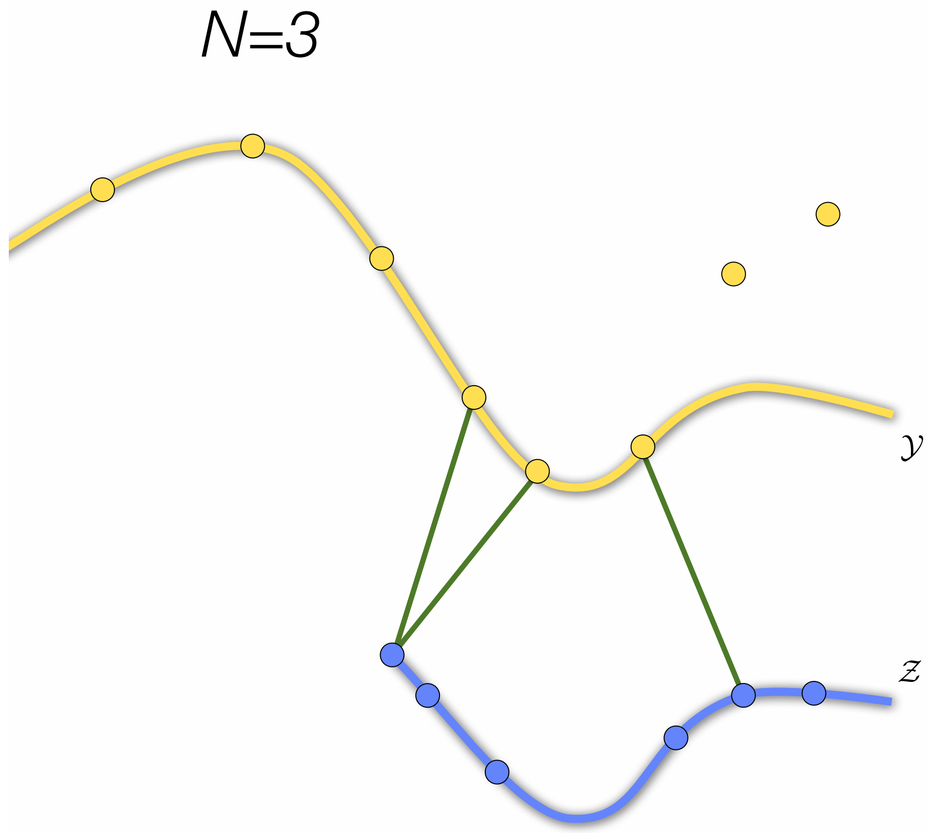
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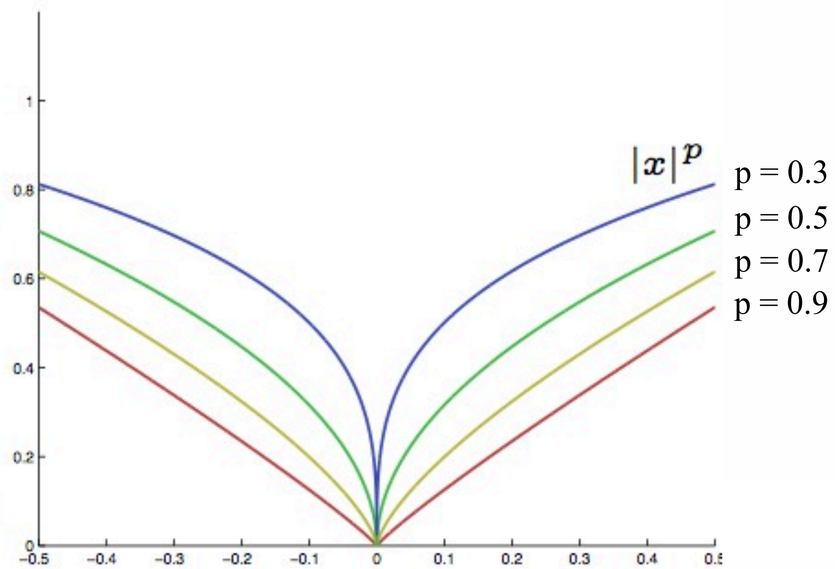


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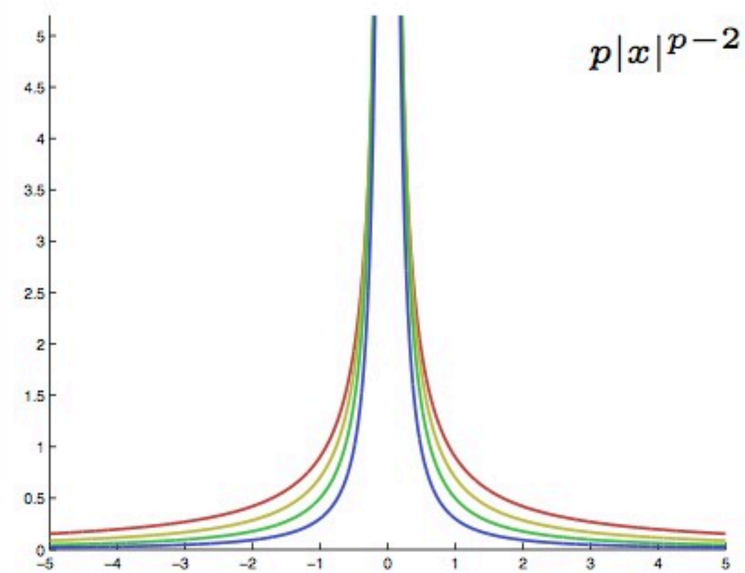
Robust Registration

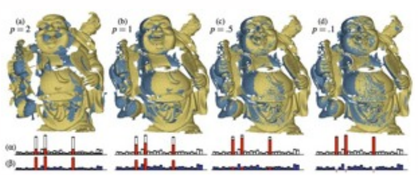
- Sparse Metrics

$$\varphi(x) = |x|^p$$



$$w(x) = p|x|^{p-2}$$



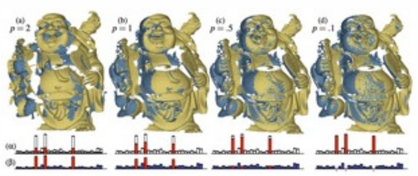


Bouaziz, Tagliasacchi, Pauly
Sparse Iterative Closest Point
SGP 2013

Robust Registration

- Sparse Metrics

$$\varphi(x) = |x|^p \qquad w(x) = p|x|^{p-2}$$

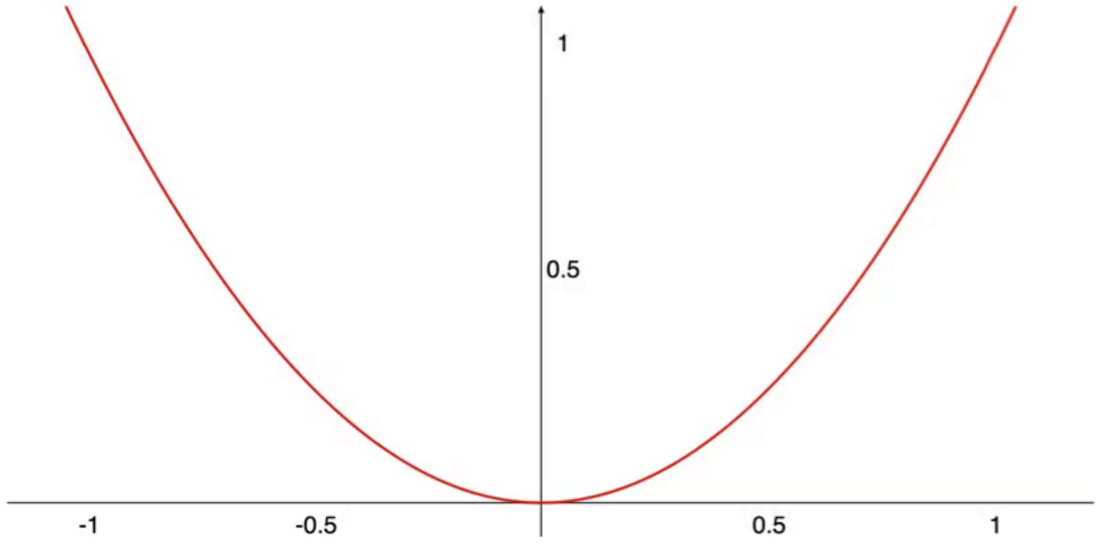


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SGP 2013

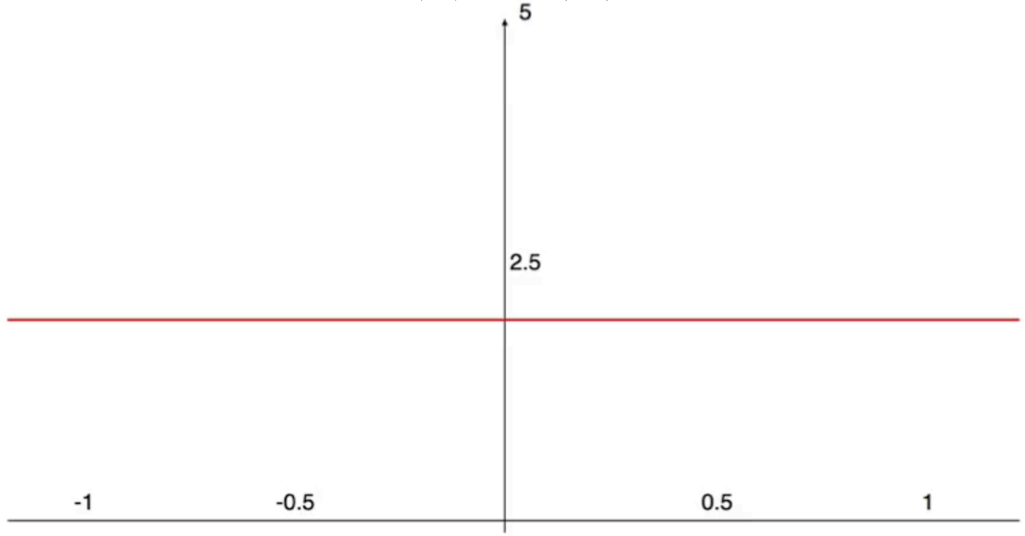
Robust Registration

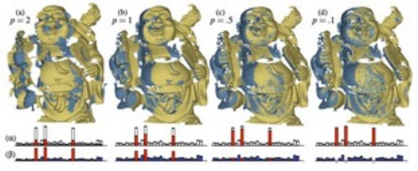
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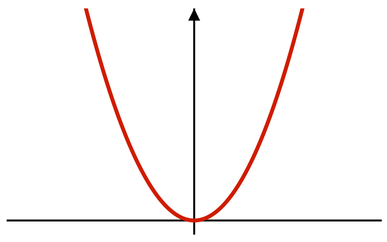




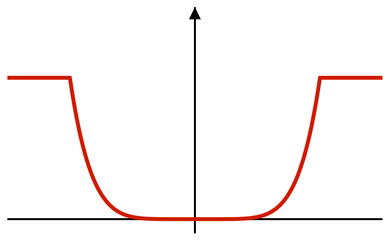
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SGP 2013

Robust Registration

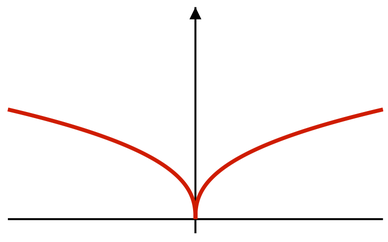
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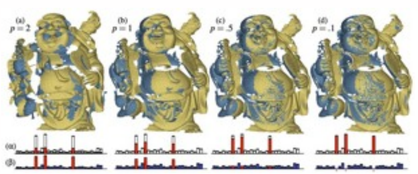


Tukey



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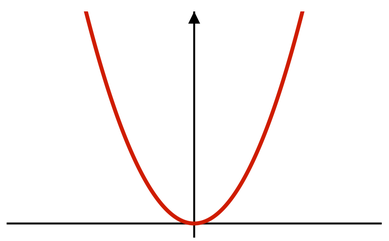


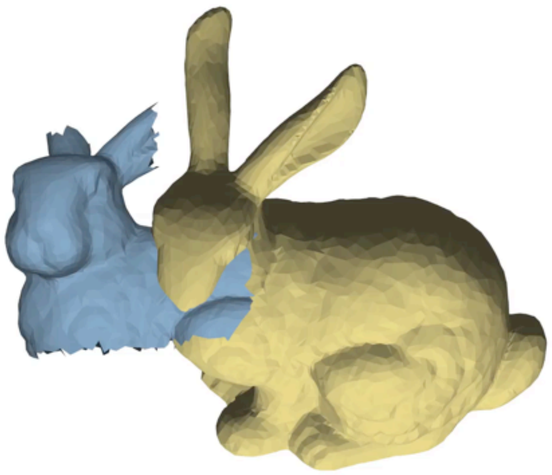


Bouaziz, Tagliasacchi, Pauly
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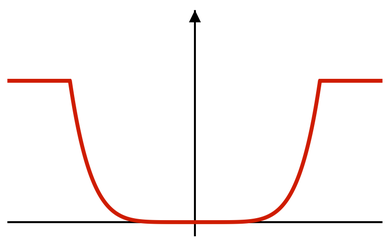
Robust Registration

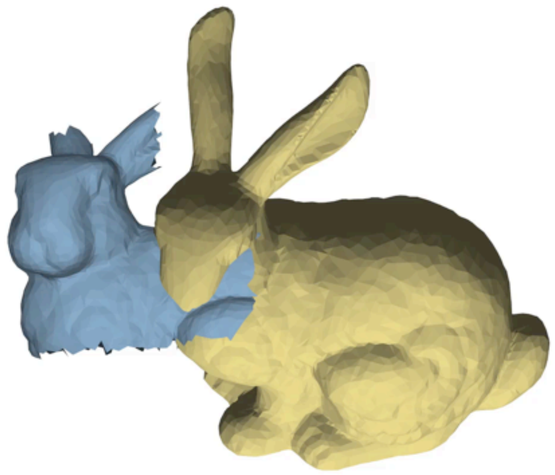
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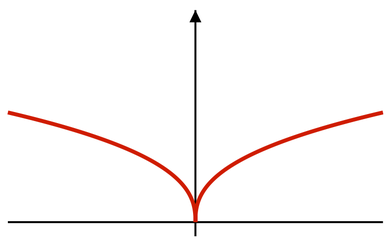


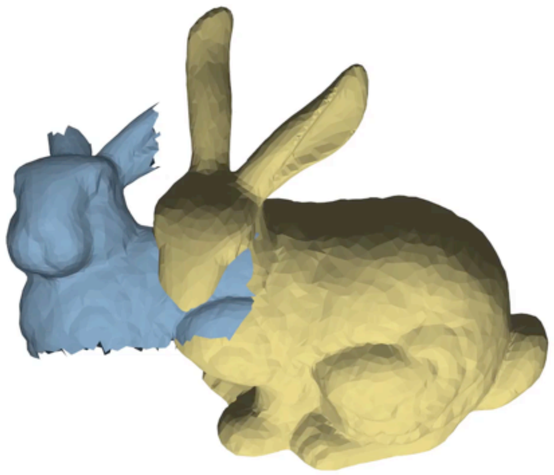
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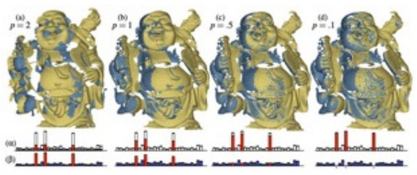




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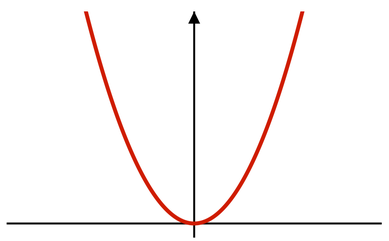


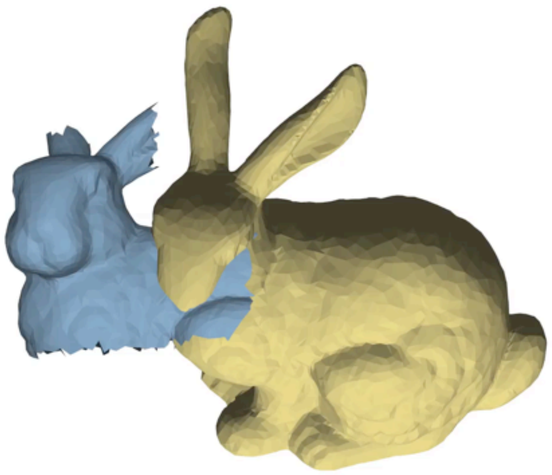


Bouaziz, Tagliasacchi, Pauly
Sparse Iterative Closest Point
SGP 2013

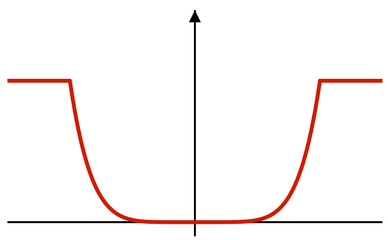
Robust Registration

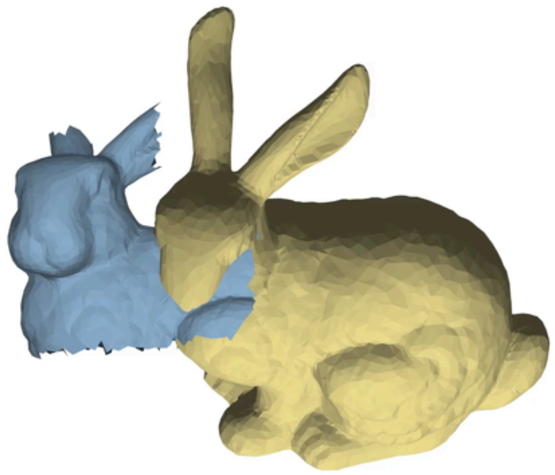
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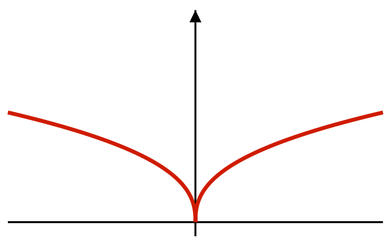


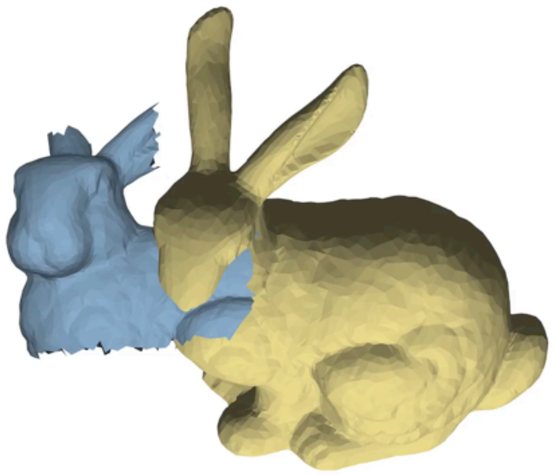
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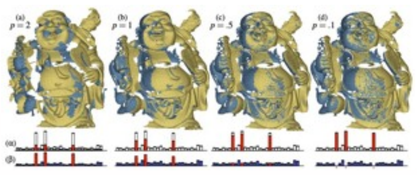




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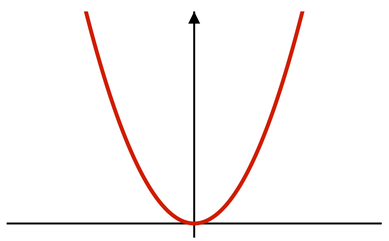


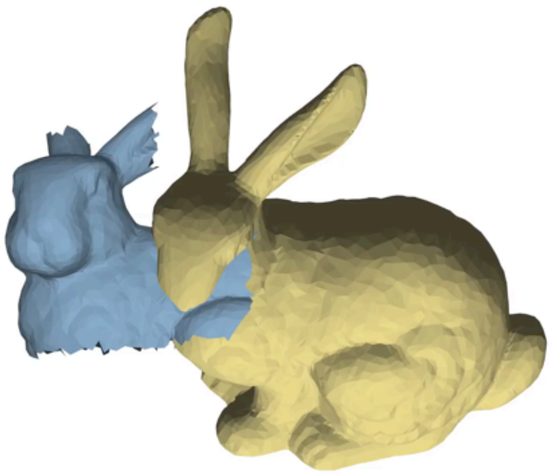


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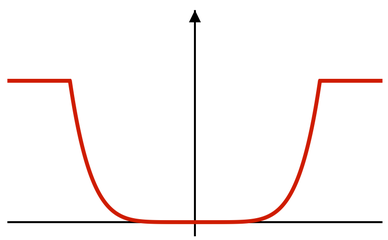
Robust Registration

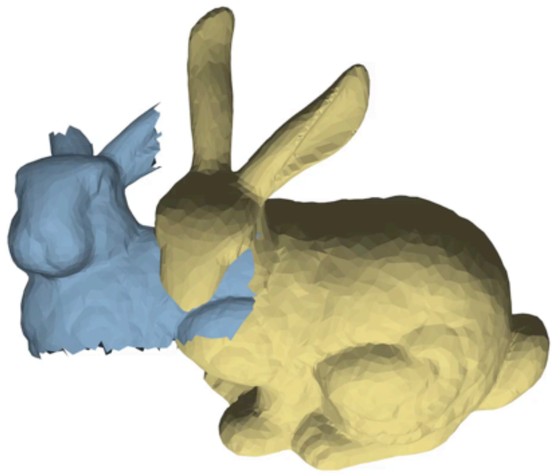
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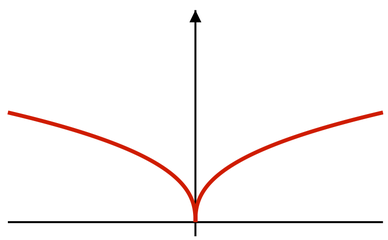


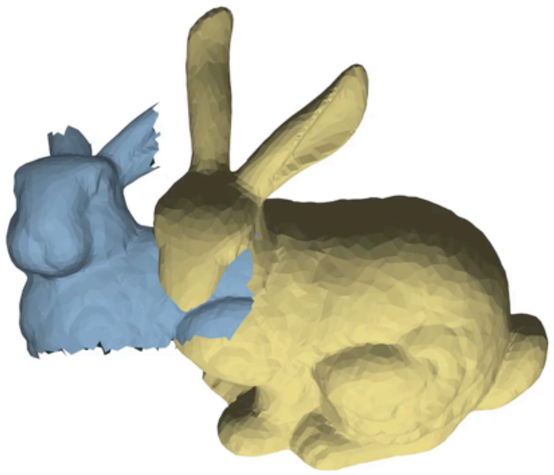
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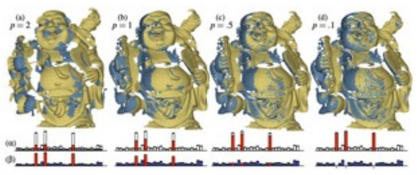




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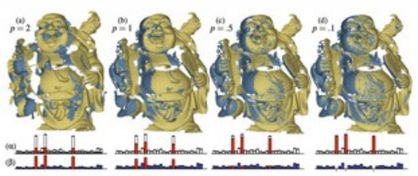






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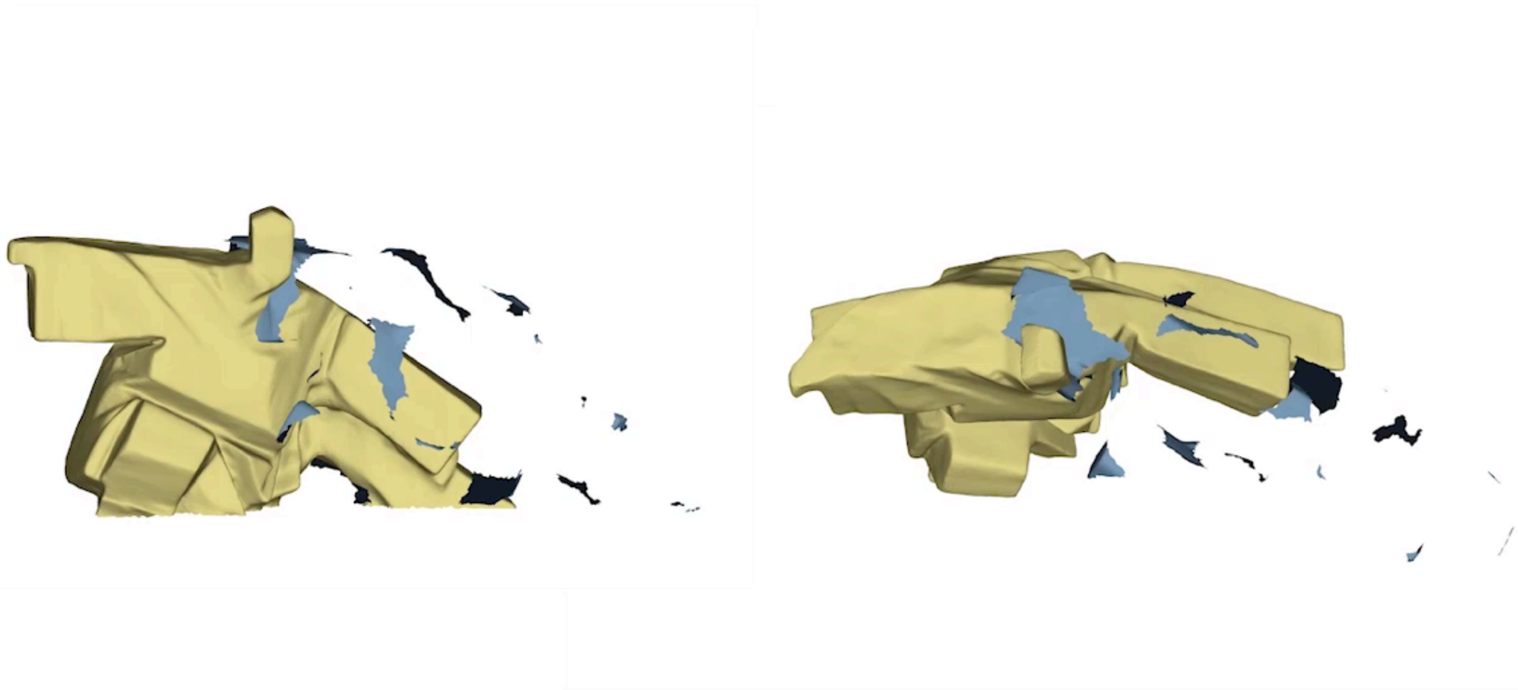
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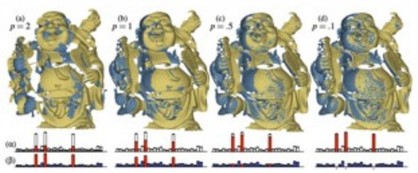


Bouaziz, Tagliasacchi, Pauly
Sparse Iterative Closest Point
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http://lgg.epfl.ch/sparseicp

Robust Registration





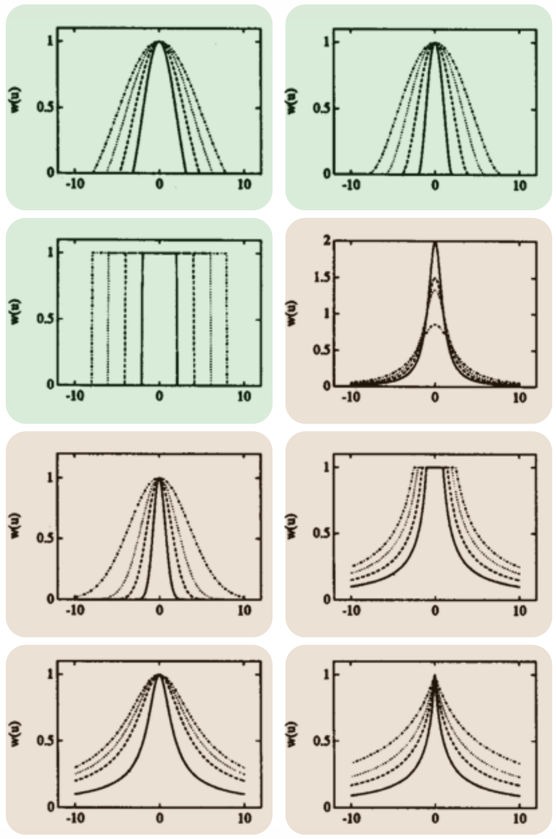
Bouaziz, Tagliasacchi, Pauly
Sparse Iterative Closest Point
SGP 2013

http://lgg.epfl.ch/sparseicp

Robust Registration - Recap

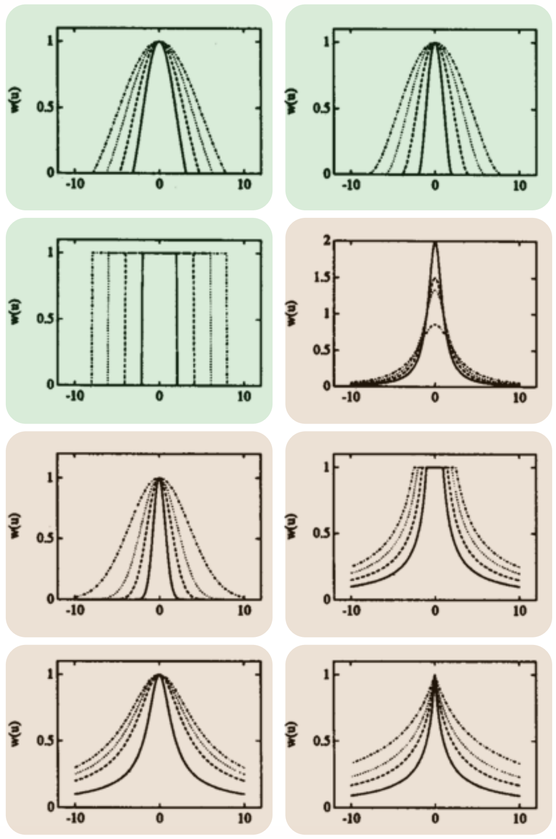
Robust Registration - Recap

Local/Global

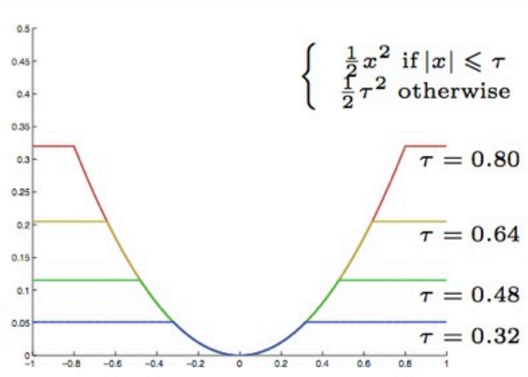


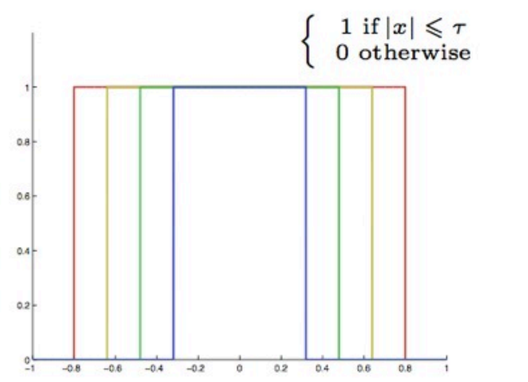
Robust Registration - Recap

Local/Global



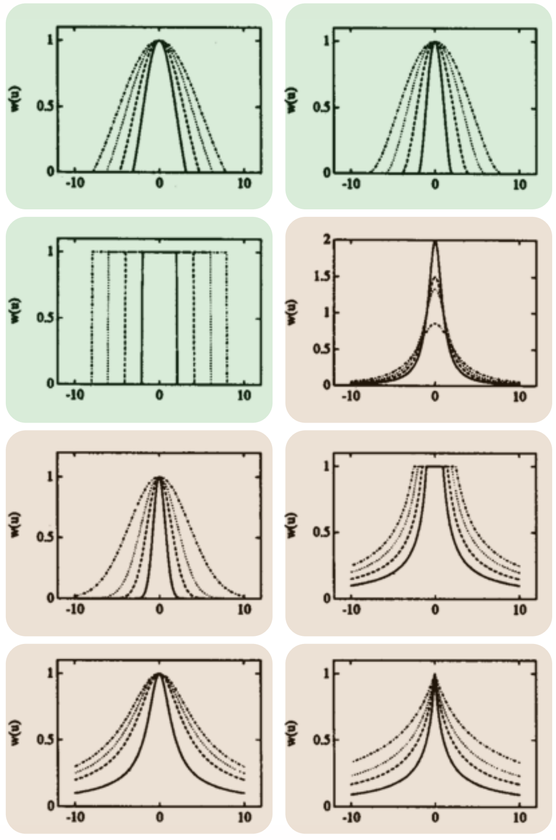
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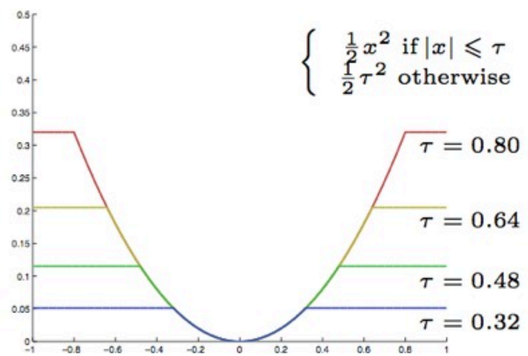


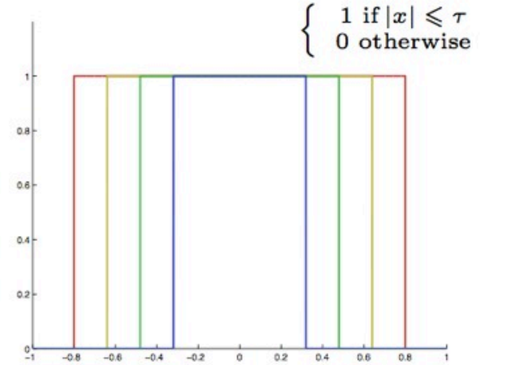
Robust Registration - Recap

Local/Global

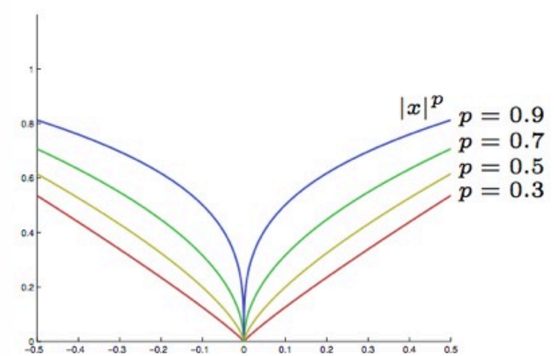


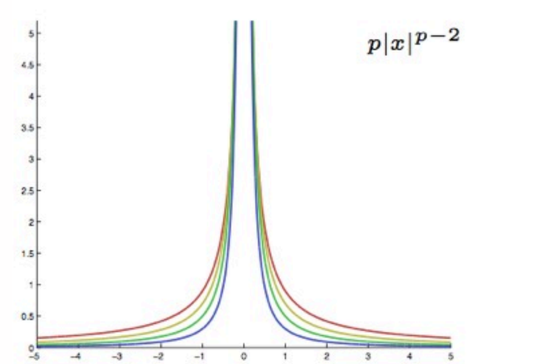
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Sparse





Overview

Introduction

- Registration
  - 3D Geometry
  - 2D Images
  - Combined 2D/3D
  - Robust Registration
  - Q&A



- Applications
  - Rigid Scanning
  - Non-rigid Modeling
  - Realtime Face Tracking

Conclusions

Q&A

Break (5min)



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Applications

Rigid Scanning

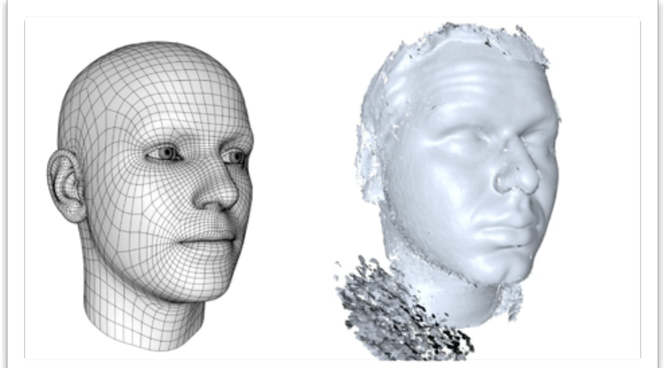
Non-Rigid Modeling

Face Tracking

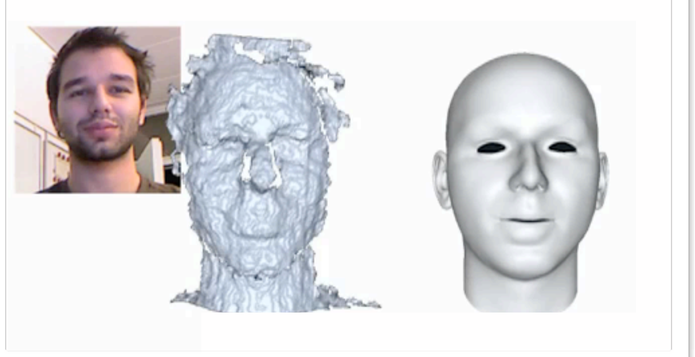
Applications

Rigid Scanning

Non-Rigid Modeling



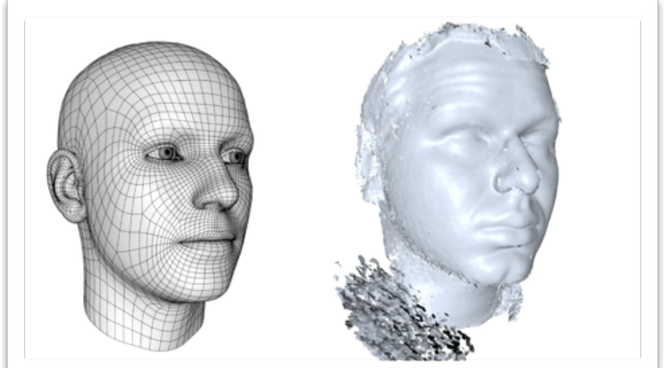
Face Tracking



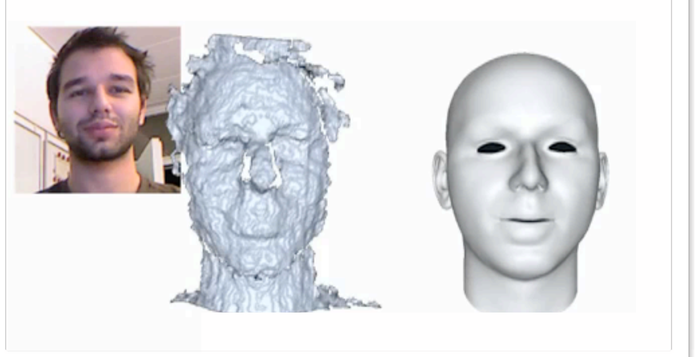
Applications

Rigid Scanning

Non-Rigid Modeling



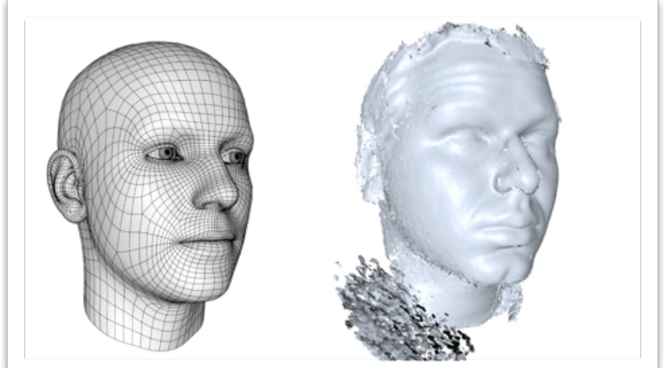
Face Tracking



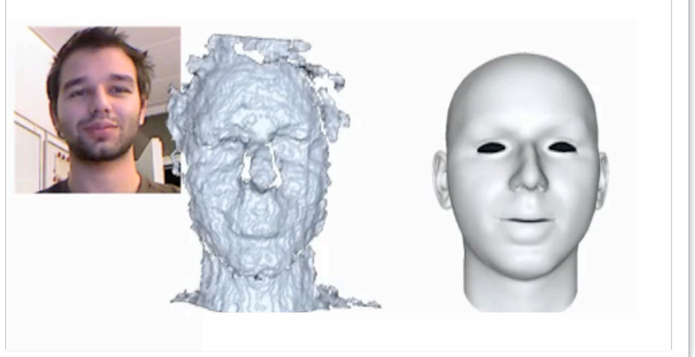
Applications

Rigid Scanning

Non-Rigid Modeling



Face Tracking



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Rigid Scanning

KSCAN3D

ImFusion

PROFACTOR®

FARO®

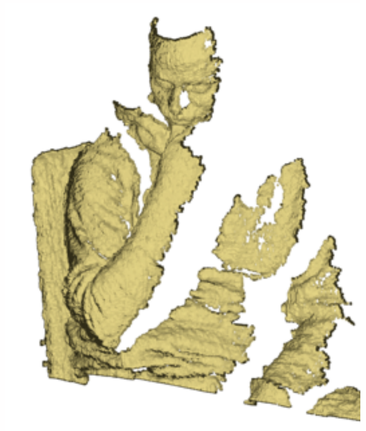
Artec Group
3D Scanning Technologies

Microsoft®

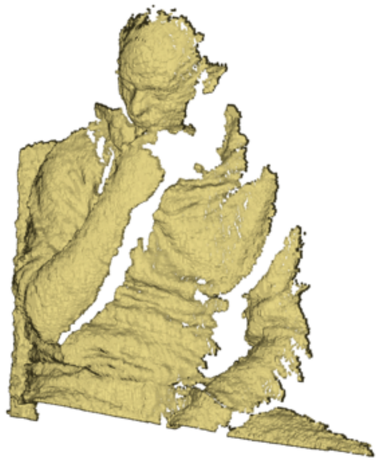
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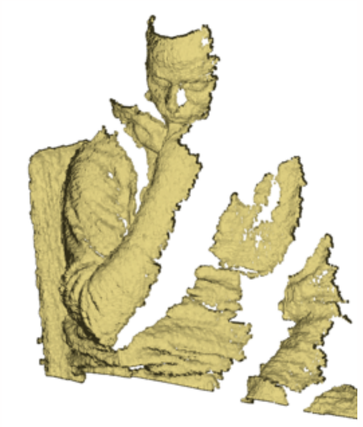
Rigid Scanning

Rigid Scanning

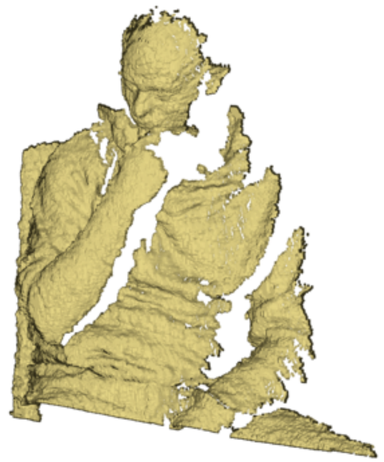


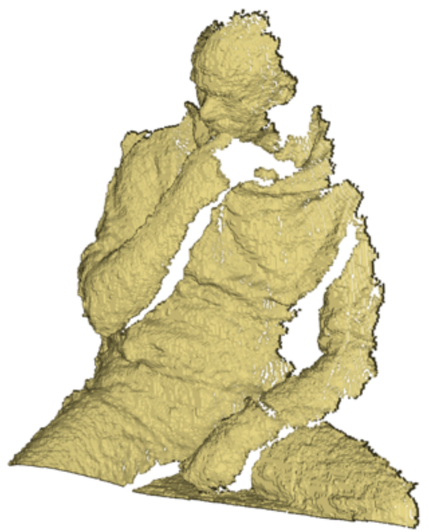
Rigid Scanning

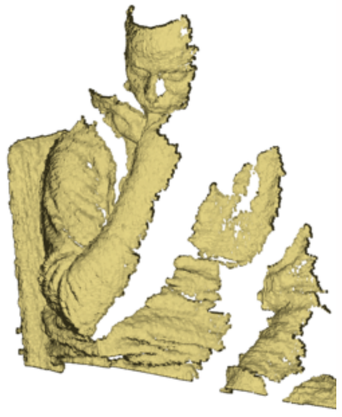




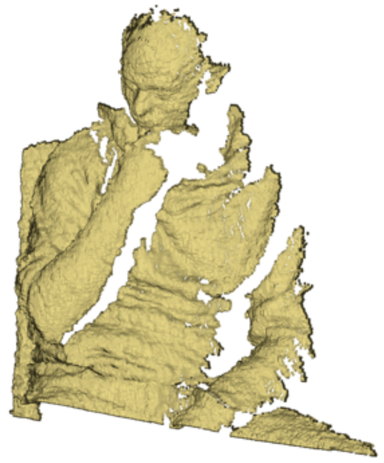
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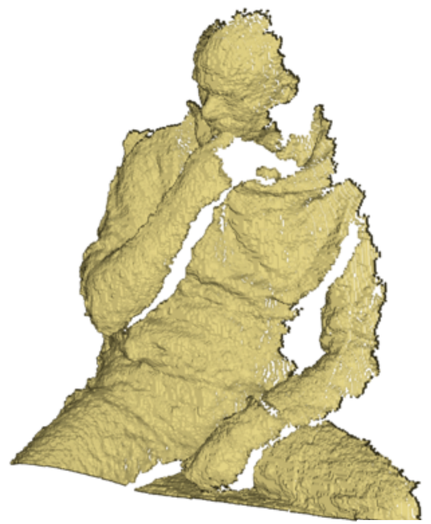


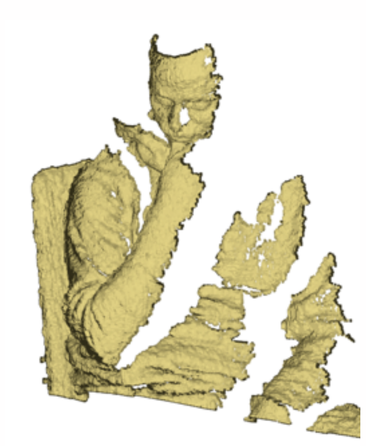




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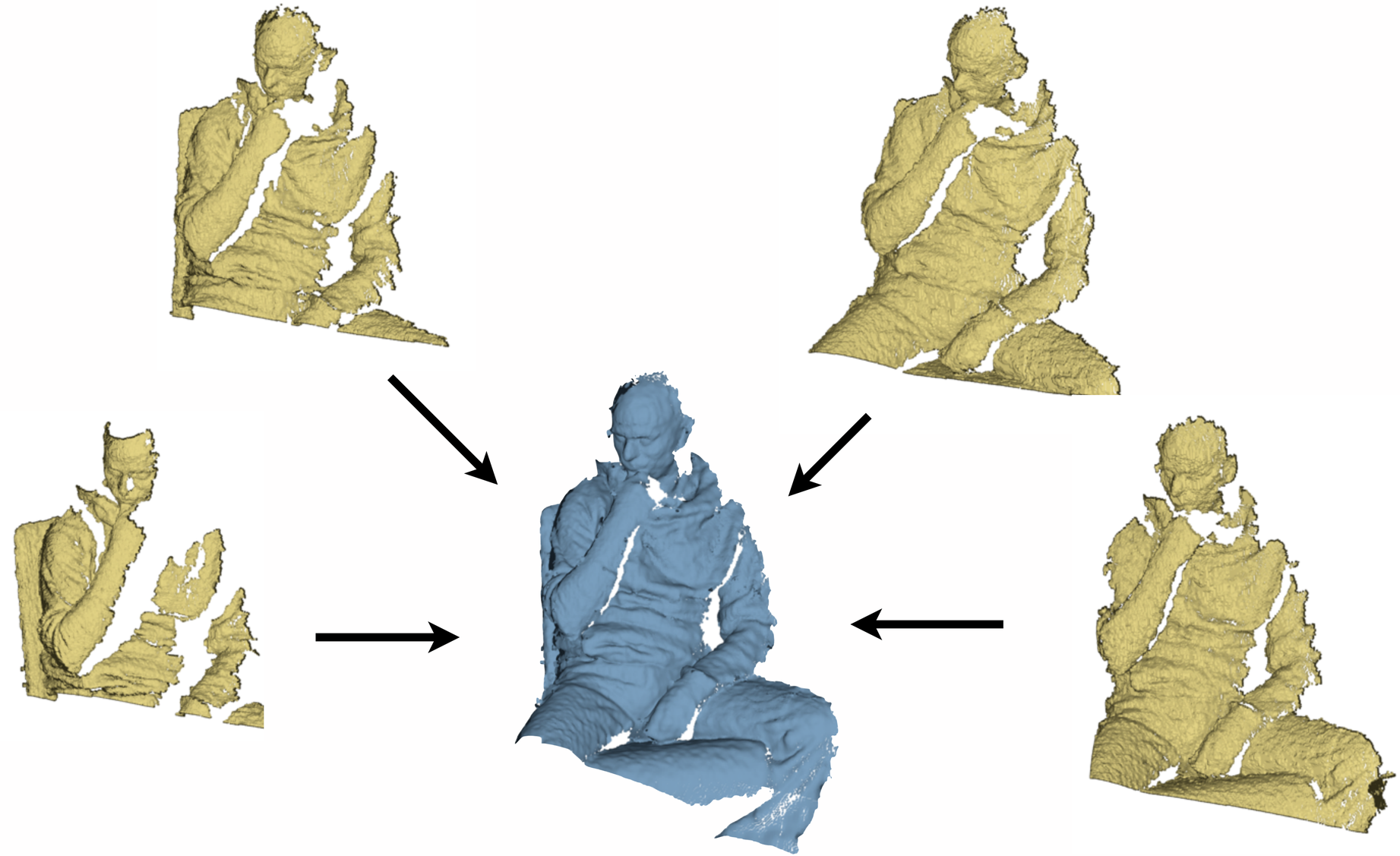








Rigid Scanning



Rigid Scanning - Pipeline



Rigid Scanning - Pipeline



Preprocessing

Depth map



Smoothing

Rigid Scanning - Pipeline



Preprocessing

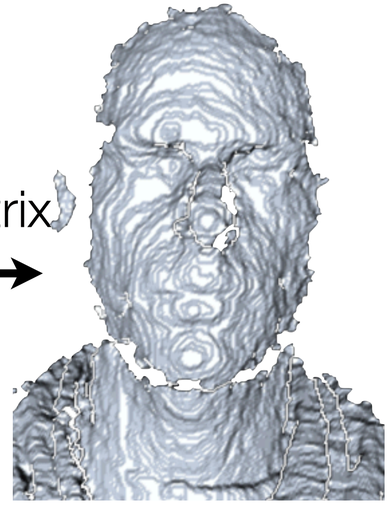
Depth map



Smoothing

Calibration Matrix

Mesh & Normals



Rigid Scanning - Pipeline



Preprocessing

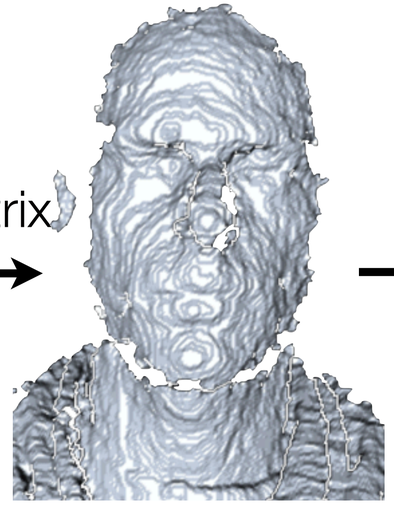
Depth map



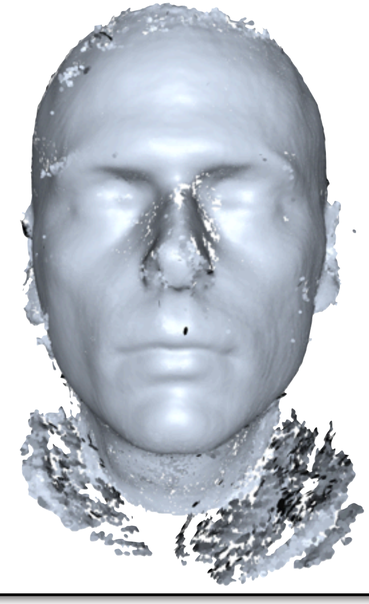
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Alignment & Accumulation



Rigid Scanning - Pipeline



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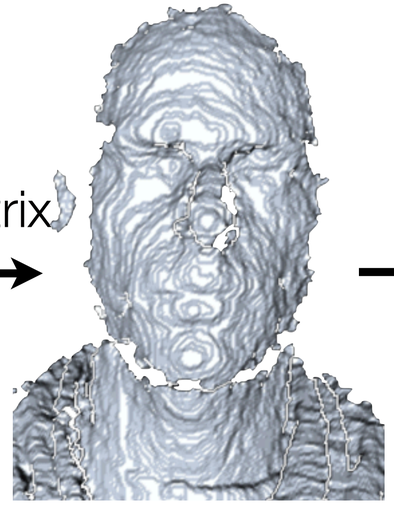
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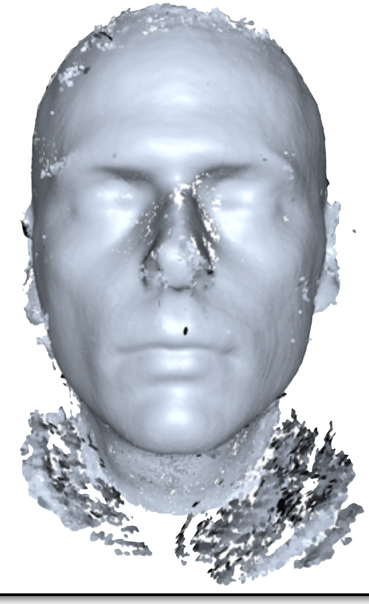
Smoothing

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Alignment & Accumulation



Rigid Scanning - Preprocessing

Iterative Smoothing







filtered



reference

Rigid Scanning - Preprocessing

Iterative Smoothing



Gaussian









filtered

reference

Rigid Scanning - Preprocessing

Iterative Smoothing



Gaussian

Bilateral











filtered

reference

Rigid Scanning - Preprocessing

Iterative Smoothing



Gaussian

Bilateral

Sparsity (TV, L0, ...)







filtered







reference

Rigid Scanning - Preprocessing

Iterative Smoothing



Gaussian

Bilateral

Sparsity (TV, L0, ...)







filtered

quality

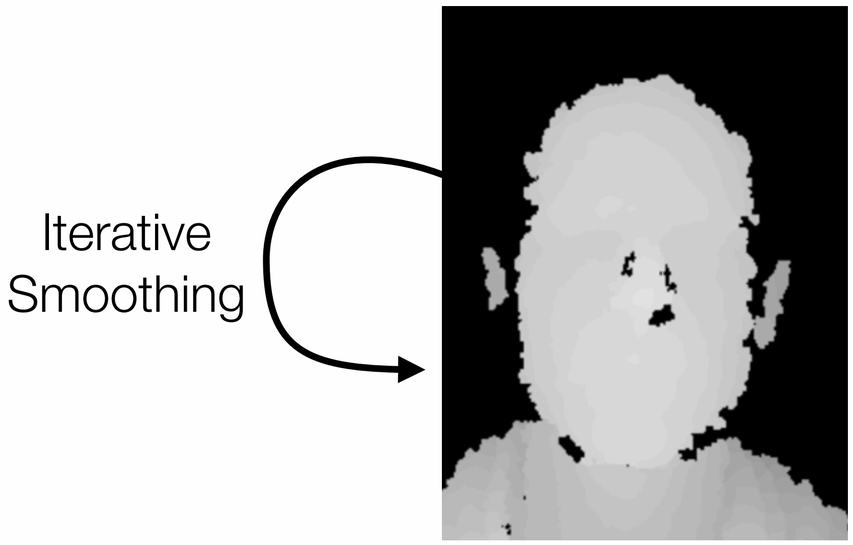


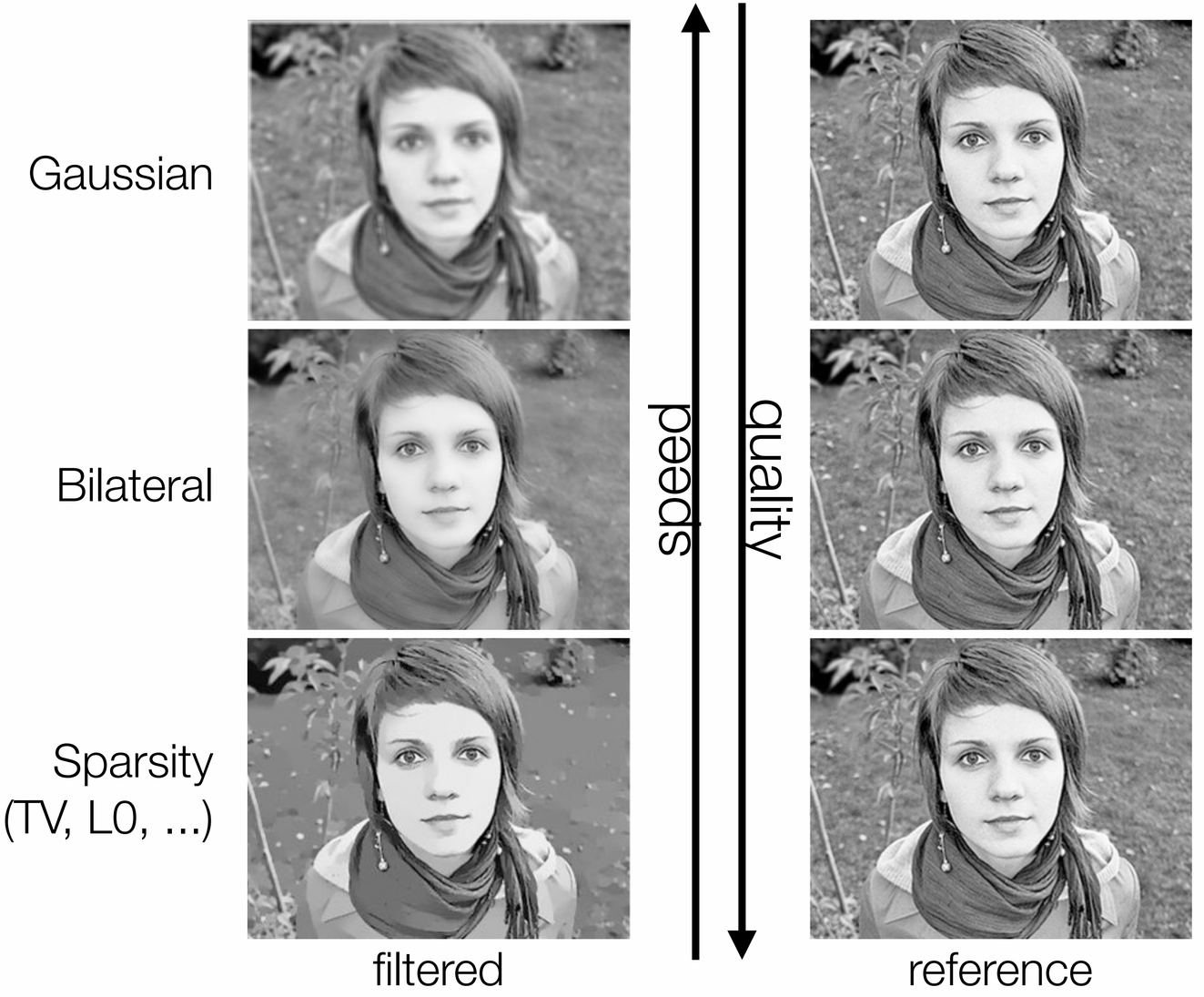




reference

Rigid Scanning - Preprocessing





Rigid Scanning - Preprocessing

Iterative Smoothing



Gaussian

Bilateral

Sparsity (TV, L0, ...)







filtered

speed

quality

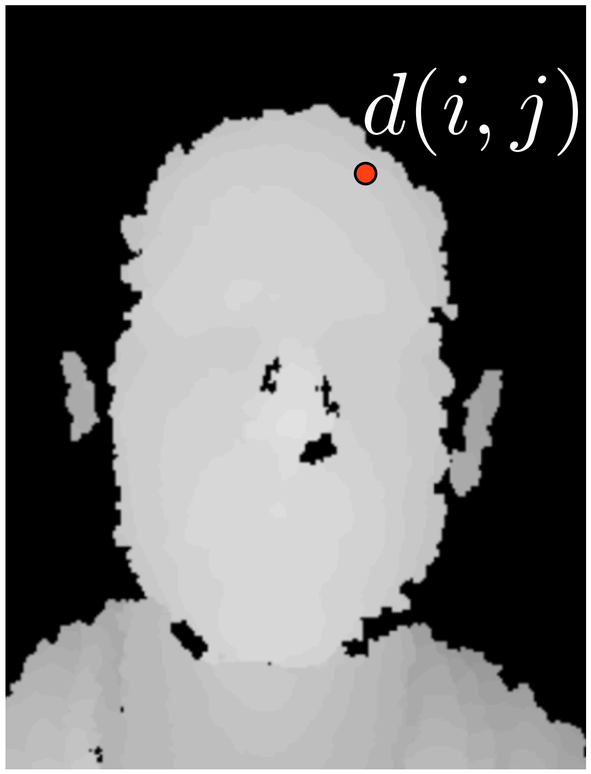




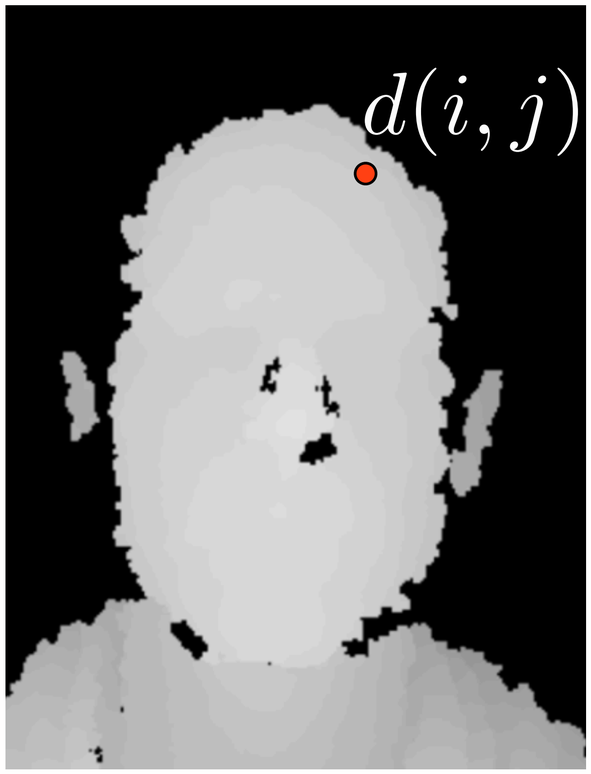


reference

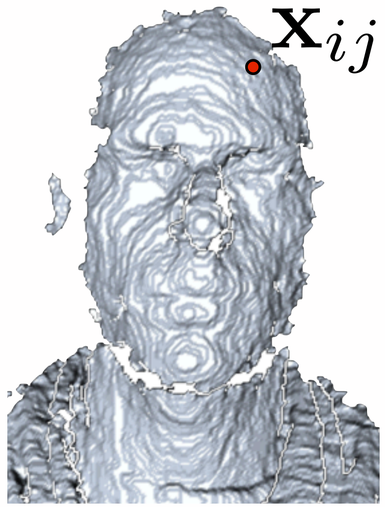
Rigid Scanning - Preprocessing



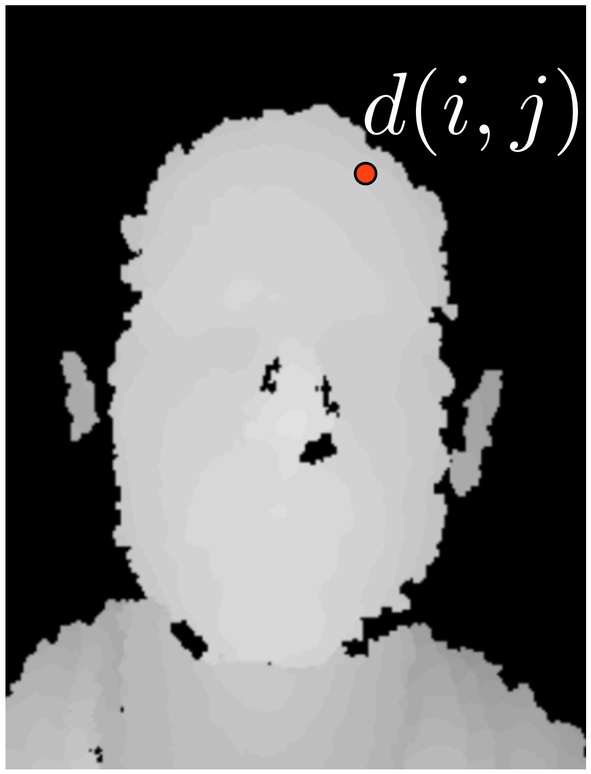
Rigid Scanning - Preprocessing



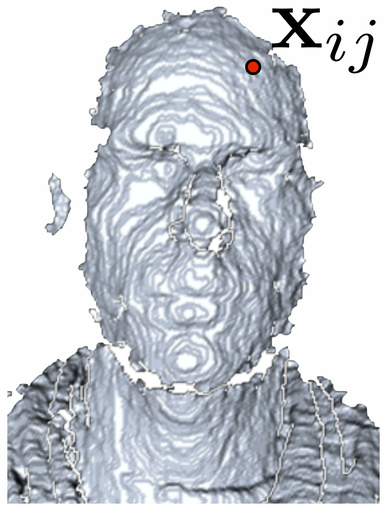
vertex positions



Rigid Scanning - Preprocessing

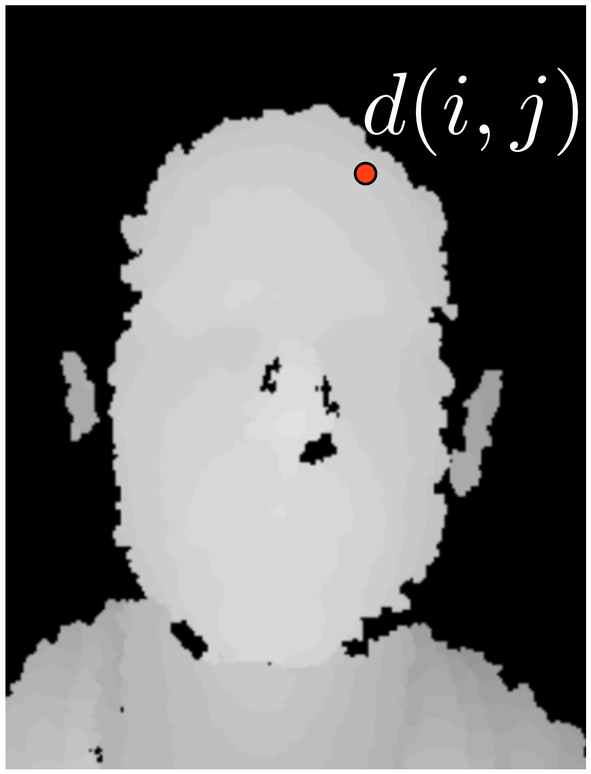


vertex positions

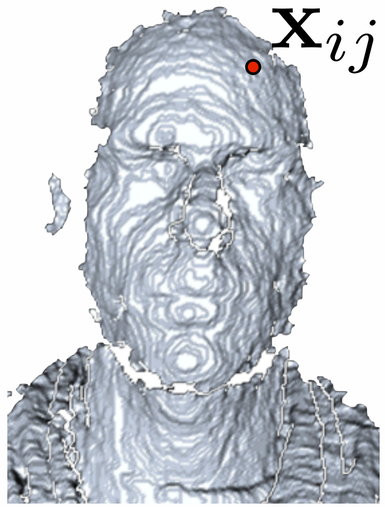


$$\mathbf{x}_{ij} = \mathbf{K}^{-1} \begin{bmatrix} i \cdot d(i,j) \\ j \cdot d(i,j) \\ d(i,j) \end{bmatrix}$$

Rigid Scanning - Preprocessing



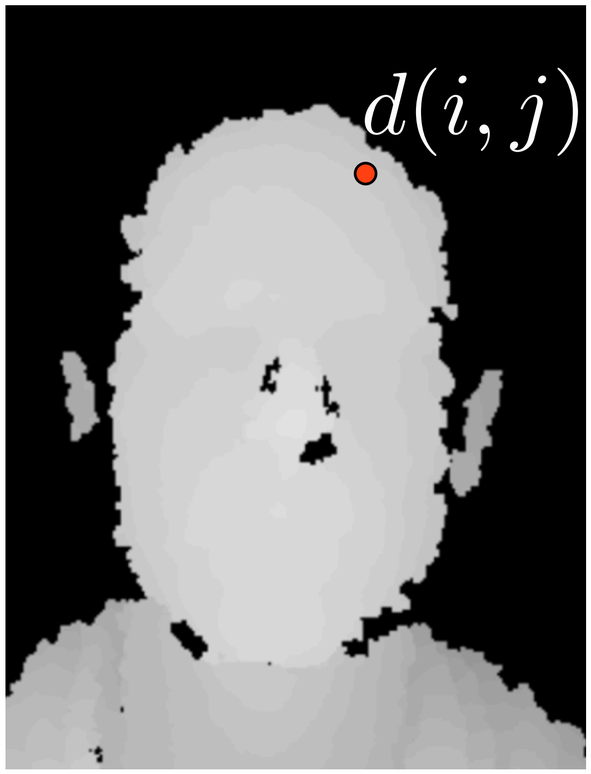
vertex positions



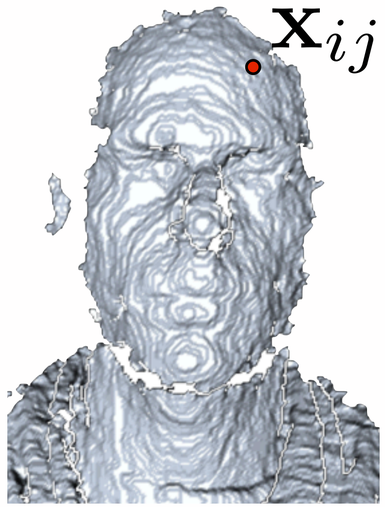
$$\mathbf{x}_{ij} = \mathbf{K}^{-1} \begin{bmatrix} i \cdot d(i,j) \\ j \cdot d(i,j) \\ d(i,j) \end{bmatrix}$$

inverse of "persp. division"

Rigid Scanning - Preprocessing



vertex positions

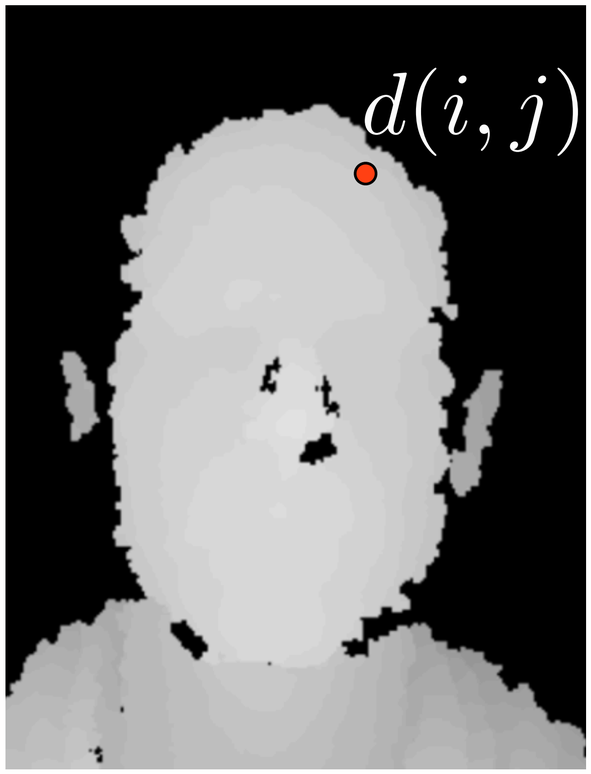


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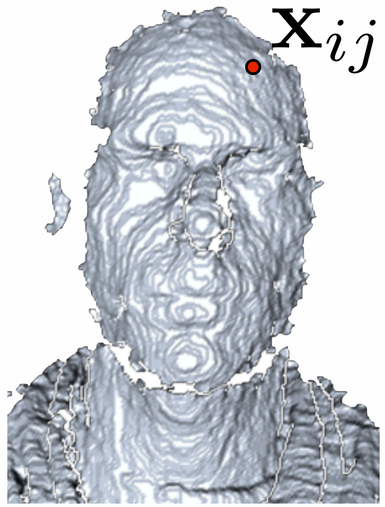
camera intrinsics (device dependent)

inverse of "persp. division"

Rigid Scanning - Preprocessing



vertex positions



$$\mathbf{x}_{ij} = \mathbf{K}^{-1} \begin{bmatrix} i \cdot d(i,j) \\ j \cdot d(i,j) \\ d(i,j) \end{bmatrix}$$

camera intrinsics (device dependent)

inverse of "persp. division"

vertex normals



finite differences (grid)

Rigid Scanning - Pipeline



Preprocessing

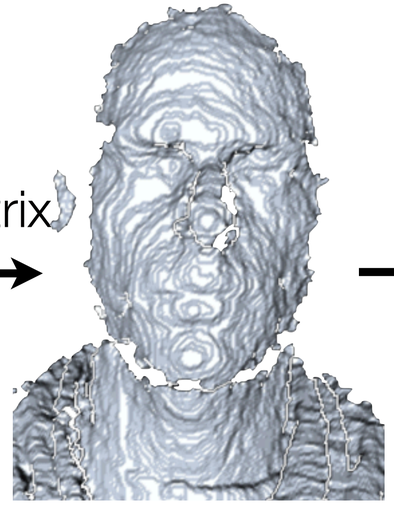
Depth map



Smoothing

Calibration Matrix

Mesh & Normals



Alignment & Accumulation



Rigid Scanning - Pipeline



Preprocessing

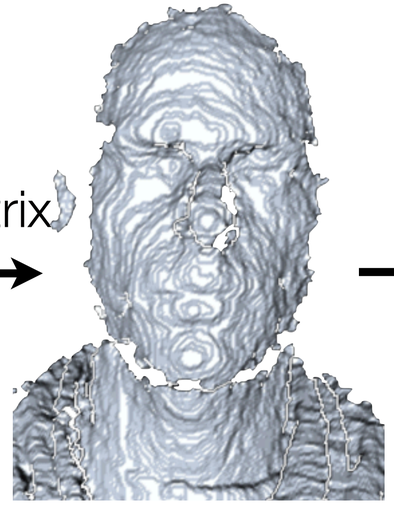
Depth map



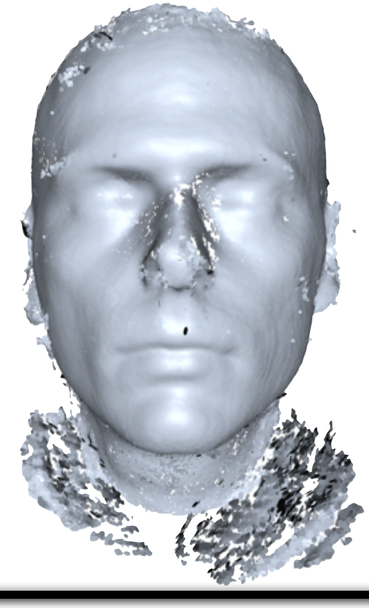
Smoothing

Calibration Matrix

Mesh & Normals

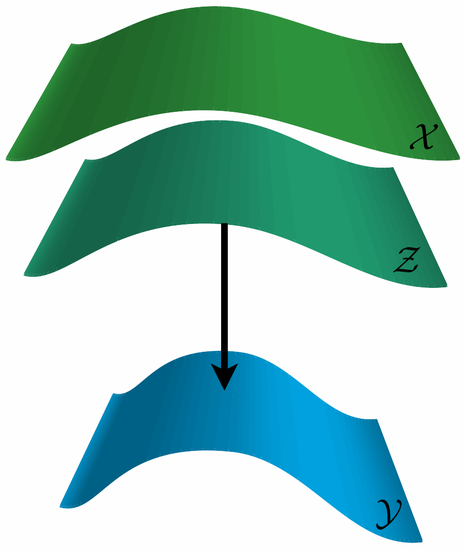


Alignment & Accumulation



Rigid Scanning - Optimization

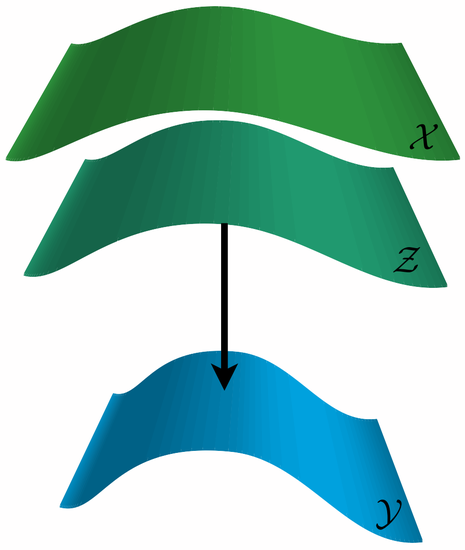
$$\underset{Z,\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$



Rigid Scanning - Optimization

$$\underset{Z,\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

matching term

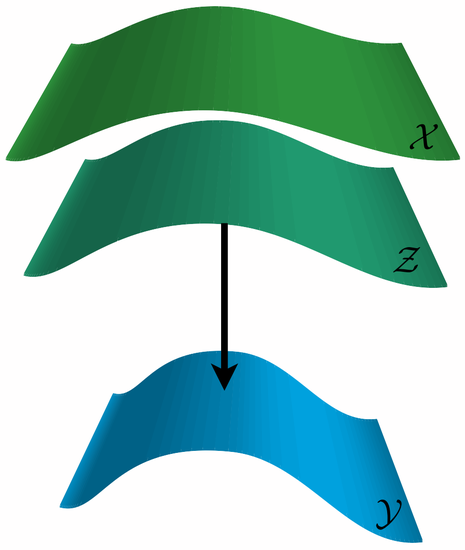


Rigid Scanning - Optimization

$$\underset{Z,\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

matching term

rigid motion



Rigid Scanning - Optimization

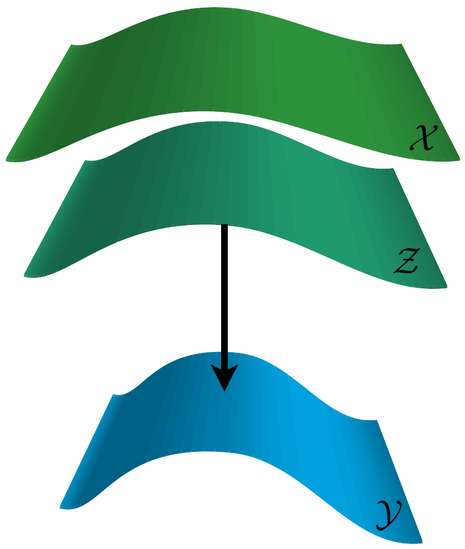
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

matching term

rigid motion

$$\Downarrow w_2 \to \infty$$

$$\arg\min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{n} \|(\mathbf{R}\mathbf{x}_i + \mathbf{t}) - P_{\mathcal{Y}}(\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$



Rigid Scanning - Optimization

$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

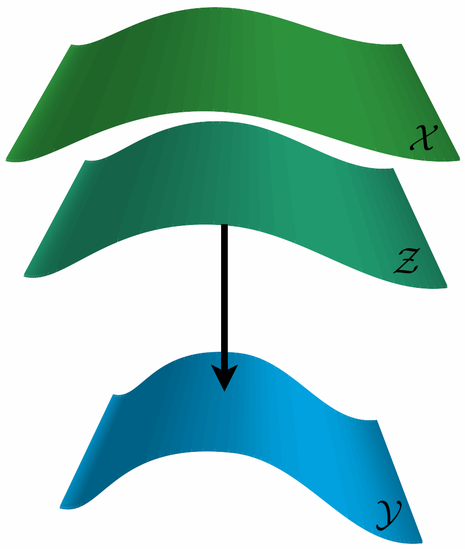
matching term

rigid motion

$$\Downarrow w_2 \to \infty$$

$$\arg\min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{n} \|(\mathbf{R}\mathbf{x}_i + \mathbf{t}) - P_{\mathcal{Y}}(\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

$$\Downarrow$$ traditional ICP



Rigid Scanning - Optimization

$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

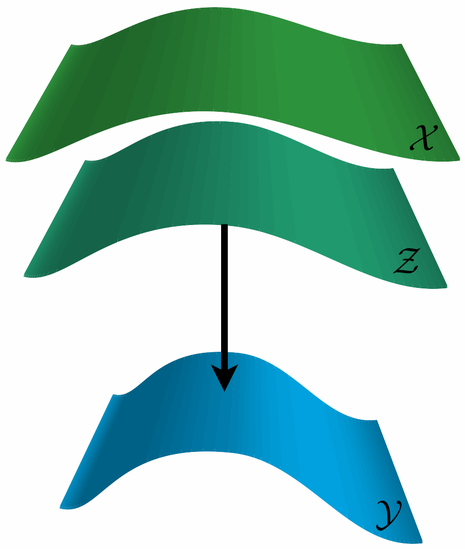
matching term

rigid motion

$$\Downarrow w_2 \to \infty$$

$$\arg\min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{n} \|(\mathbf{R}\mathbf{x}_i + \mathbf{t}) - P_{\mathcal{Y}}(\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

$$\Downarrow$$ traditional ICP



Step 1: find correspondences

Step 2: find best rigid transformation

Rigid Scanning - Optimization

$$\underset{Z,\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Rigid Scanning - Optimization

$$\underset{Z,\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

$$\Downarrow$$

$$\underset{Z^{t+1},\tilde{\mathbf{R}},\tilde{\mathbf{t}}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t\mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

Rigid Scanning - Optimization

$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

$$\arg\min_{Z^{t+1},\tilde{\mathbf{R}},\tilde{\mathbf{t}}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t\mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

use previous iteration

Rigid Scanning - Optimization

$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

$$\Downarrow$$

$$\arg\min_{Z^{t+1},\tilde{\mathbf{R}},\tilde{\mathbf{t}}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^{n} ||\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t\mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})||_2^2$$

Rigid Scanning - Optimization

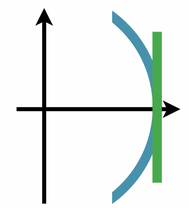
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

$$\Downarrow$$

$$\arg\min_{Z^{t+1},\tilde{\mathbf{R}},\tilde{\mathbf{t}}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^{n} ||\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t\mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})||_2^2$$

linearize rotation matrix

$$\mathbf{R} \approx \tilde{\mathbf{R}} = \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix}$$



Rigid Scanning - Optimization

$$\underset{Z,\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Rigid Scanning - Optimization

$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

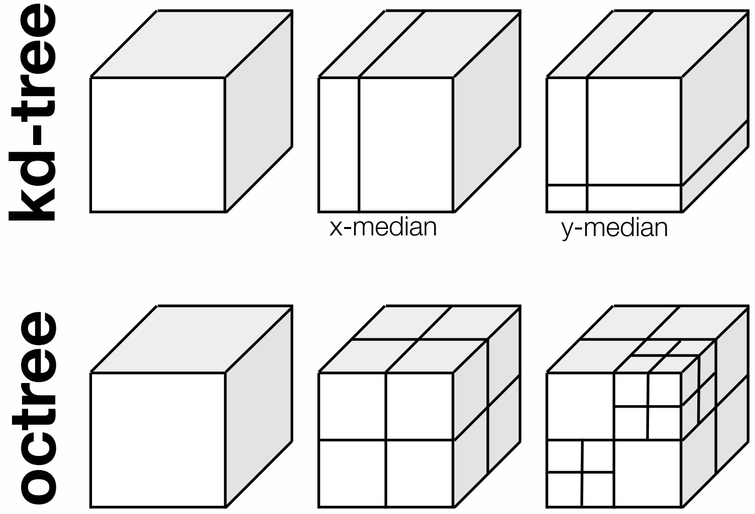
Acceleration Data Structures

Projective Lookup

Rigid Scanning - Optimization

$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Acceleration Data Structures

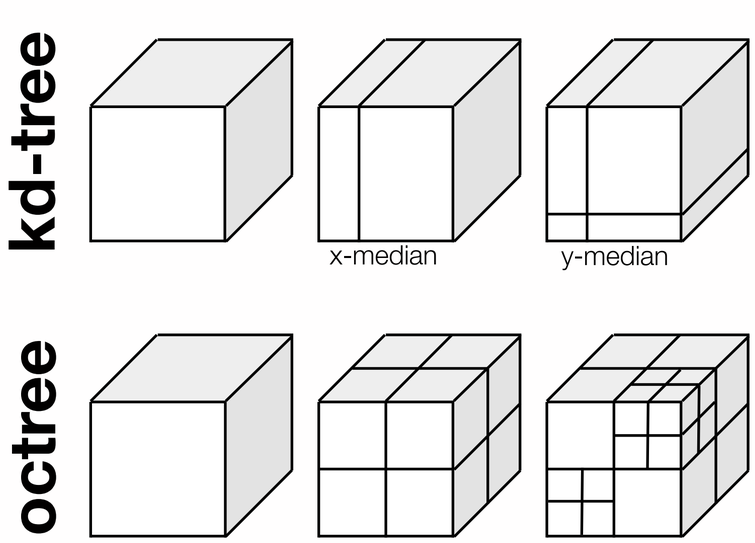


Projective Lookup

Rigid Scanning - Optimization

$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Acceleration Data Structures



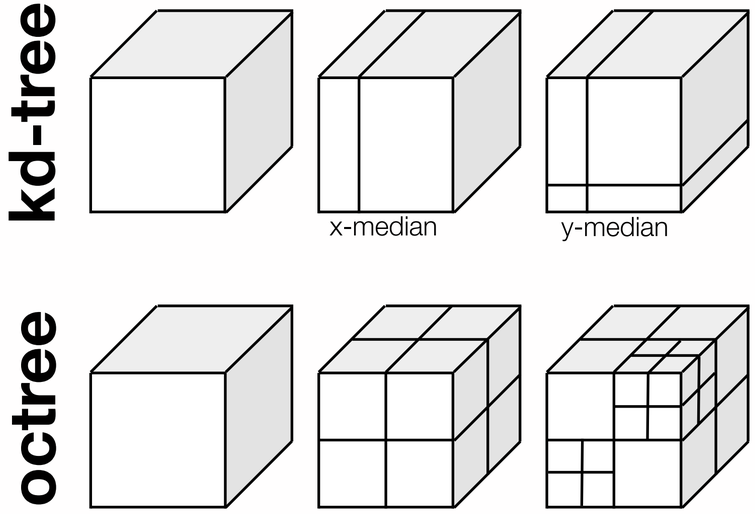
build: $O(n \log n)$, each query: $O(\log n)$

Projective Lookup

Rigid Scanning - Optimization

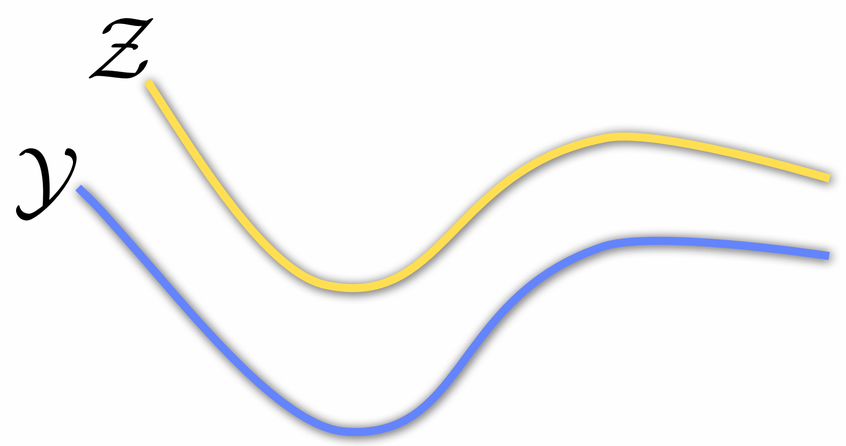
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Acceleration Data Structures



build: $O(n \log n)$, each query: $O(\log n)$

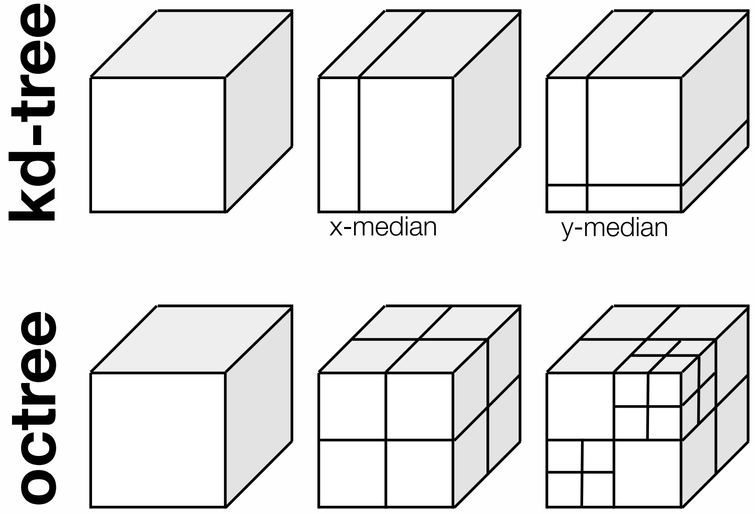
Projective Lookup



Rigid Scanning - Optimization

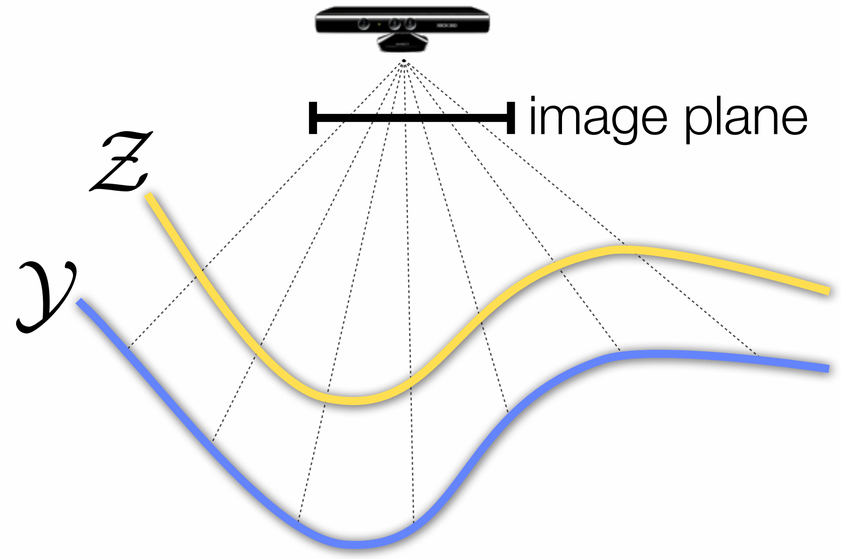
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Acceleration Data Structures



build: $O(n \log n)$, each query: $O(\log n)$

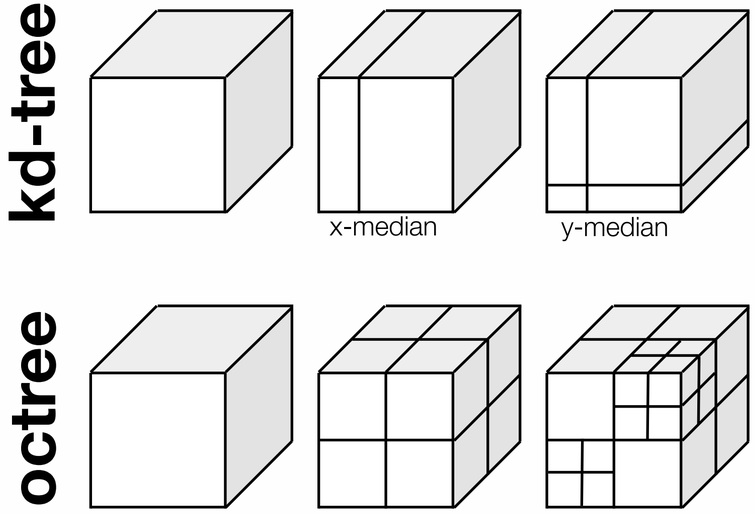
Projective Lookup



Rigid Scanning - Optimization

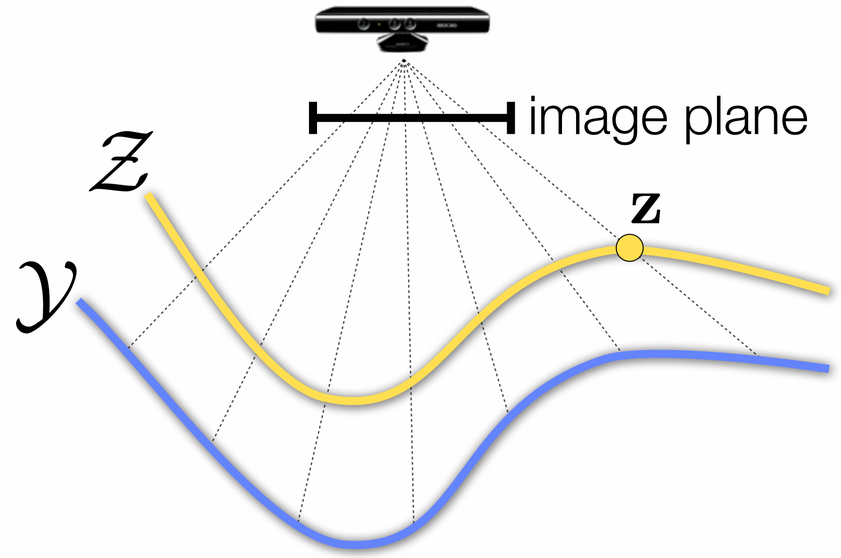
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Acceleration Data Structures



build: $O(n \log n)$, each query: $O(\log n)$

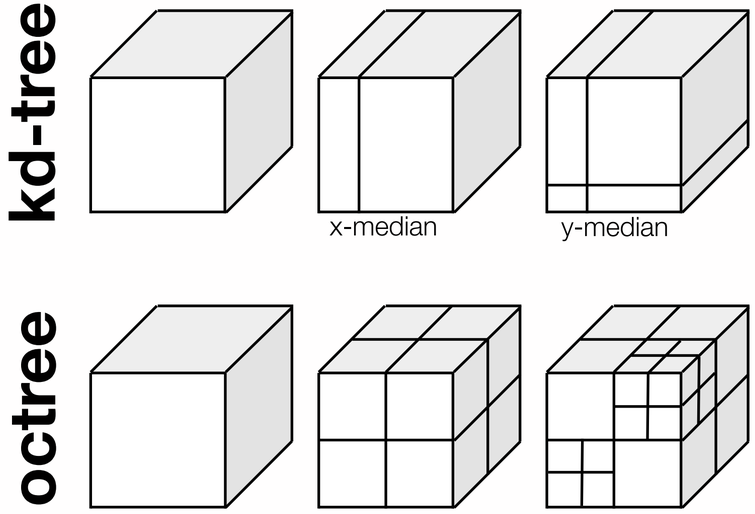
Projective Lookup



Rigid Scanning - Optimization

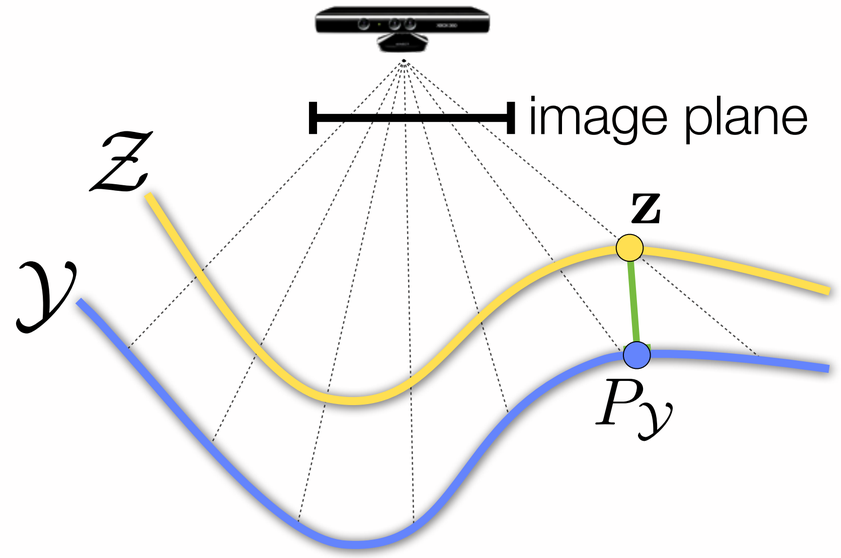
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Acceleration Data Structures



build: $O(n \log n)$, each query: $O(\log n)$

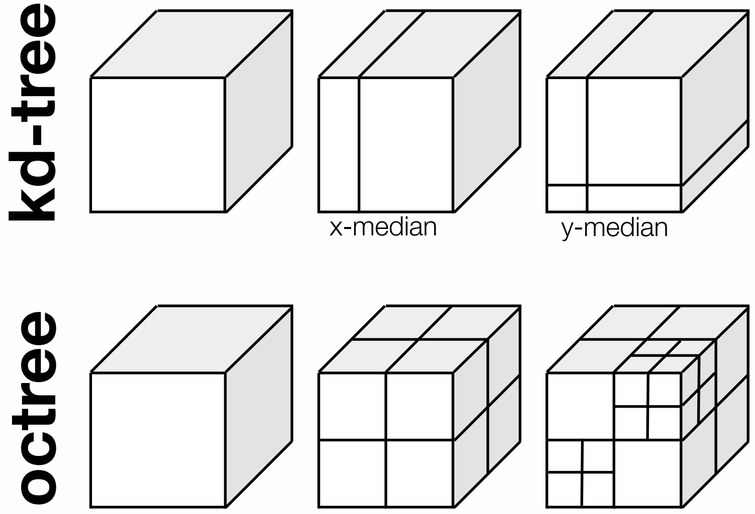
Projective Lookup



Rigid Scanning - Optimization

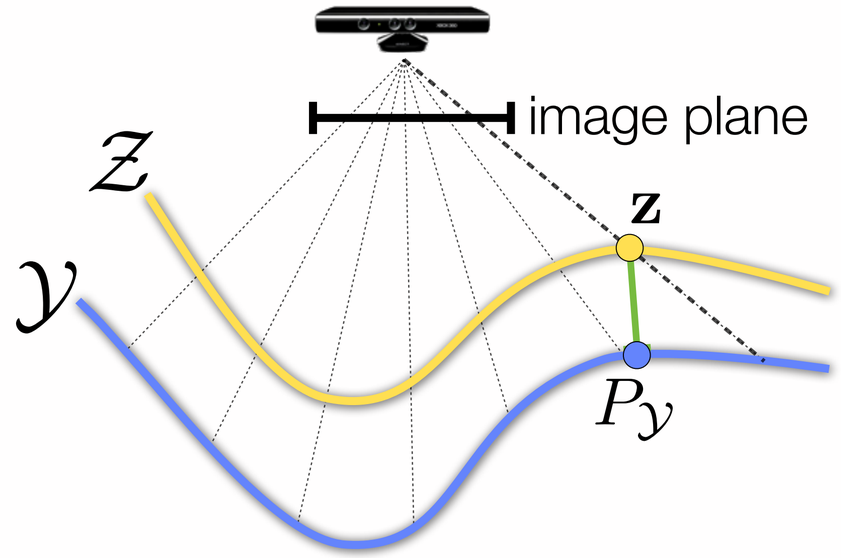
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Acceleration Data Structures



build: $O(n \log n)$, each query: $O(\log n)$

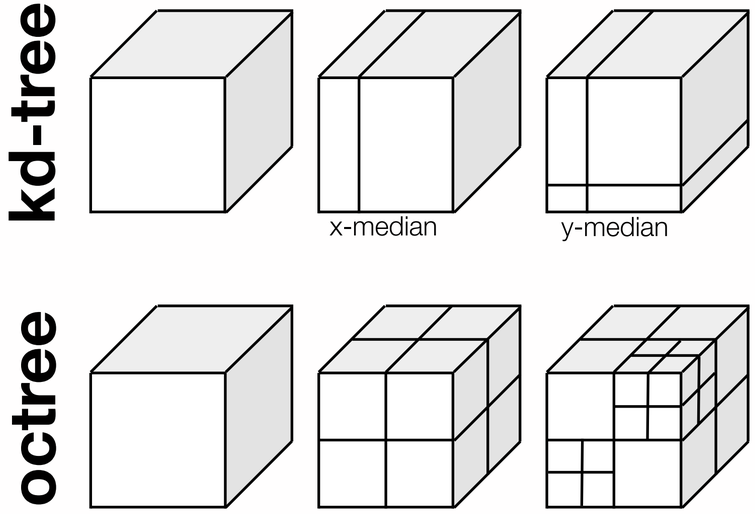
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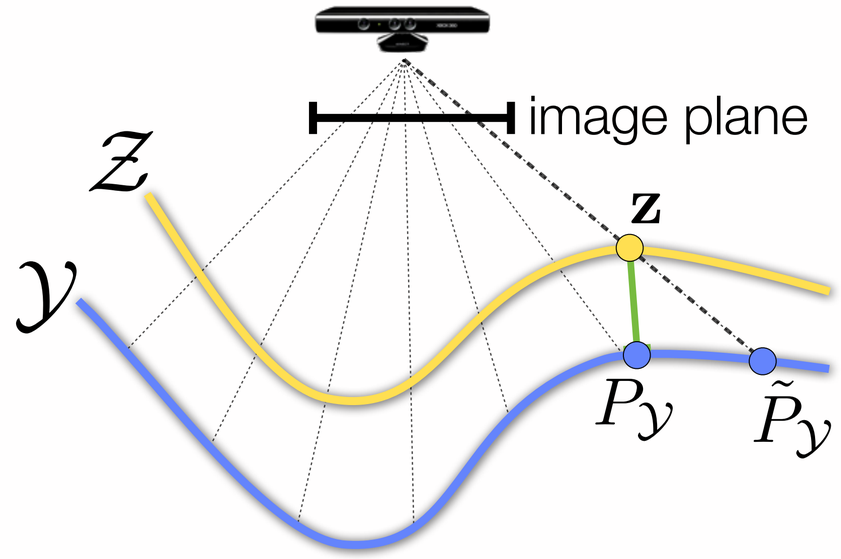
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

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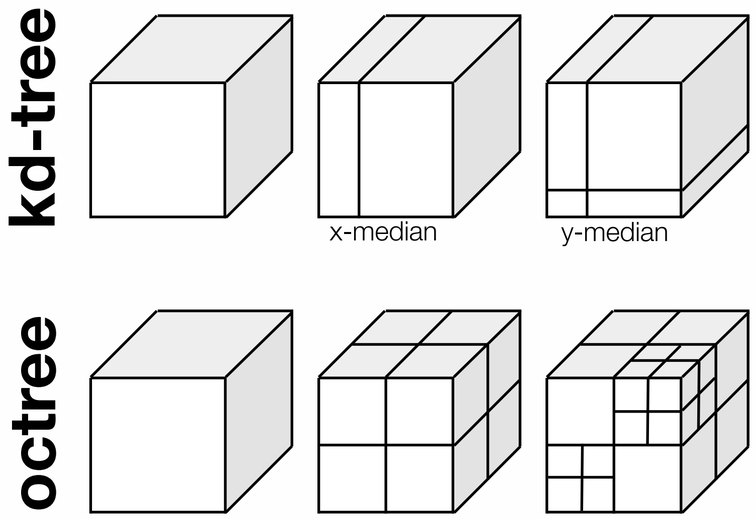
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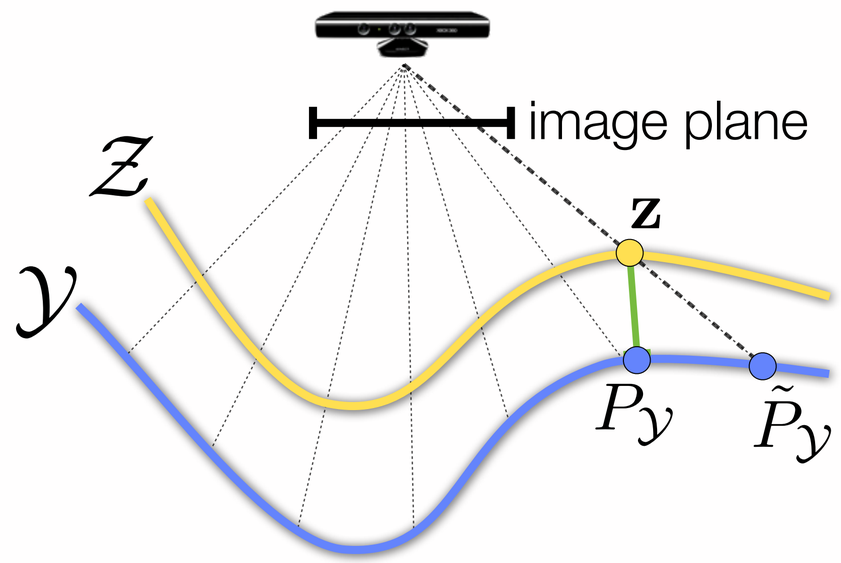
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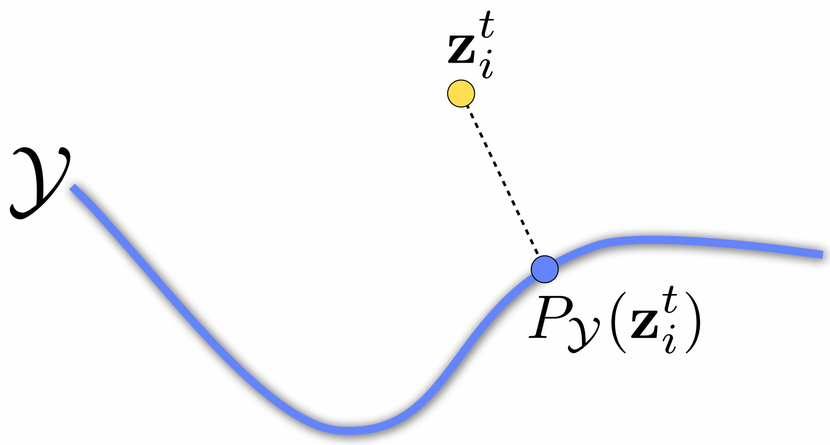
Projective Lookup

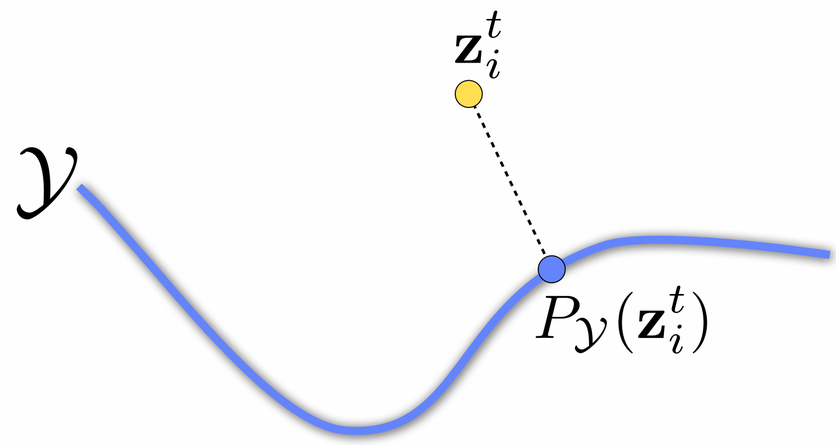


build: $O(1)$, each query: $O(1)$

Rigid Scanning - Optimization

$$\underset{Z,\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

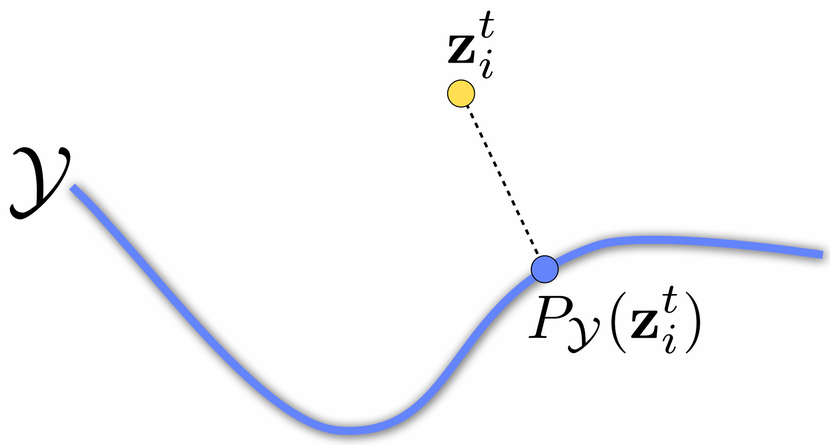




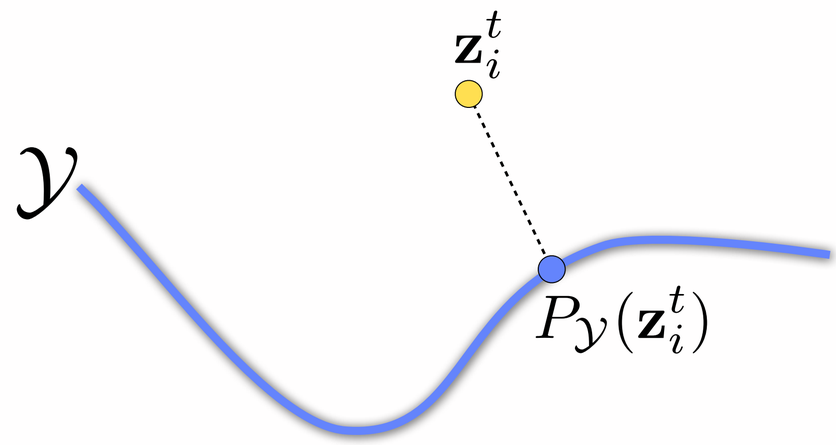
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Point-to-Point



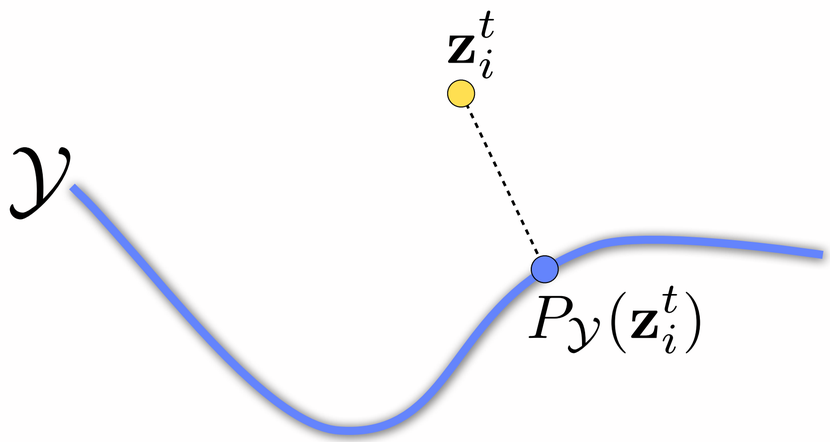
$$\|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2$$



Rigid Scanning - Optimization

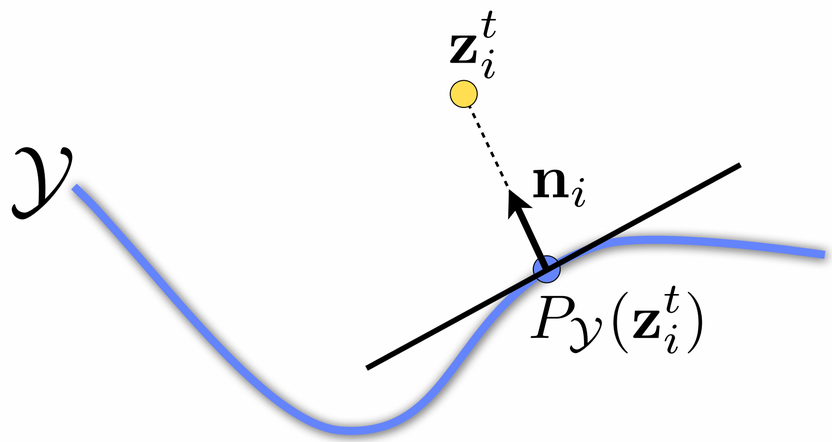
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Point-to-Point



$$\|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2$$

Point-to-Plane

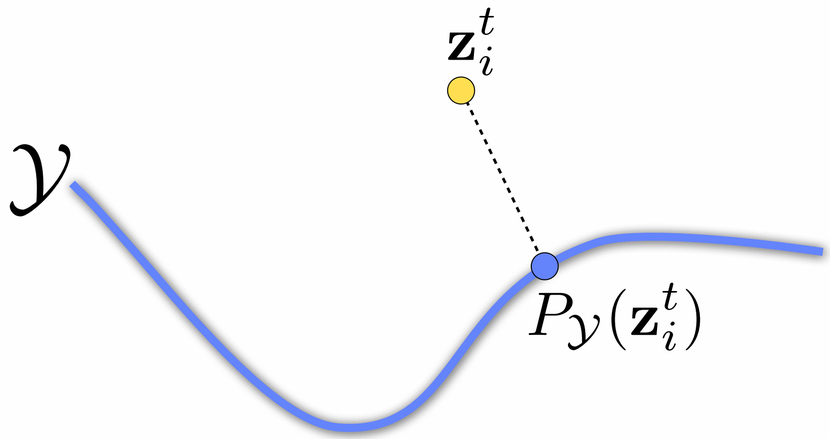


$$\langle \mathbf{n}_i, \mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t) \rangle^2$$

Rigid Scanning - Optimization

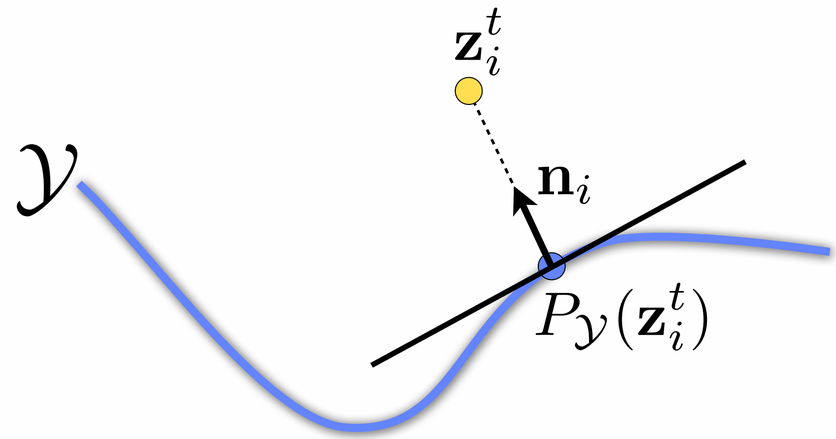
$$\arg\min_{Z,\mathbf{R},\mathbf{t}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Point-to-Point



$$\|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2$$

Point-to-Plane



$$\langle \mathbf{n}_i, \mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t) \rangle^2$$

Pottmann, Huang, Yang, Hu: **Geometry and convergence analysis of algorithms for registration of 3D shapes**, IJCV'11

Rigid Scanning - Accumulation

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Surface

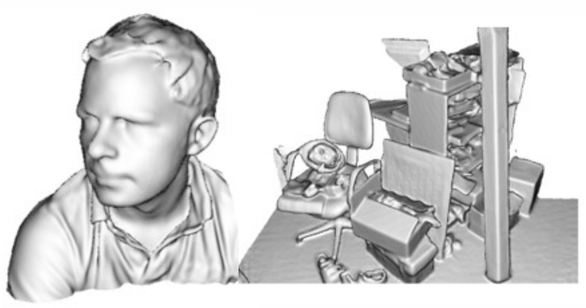


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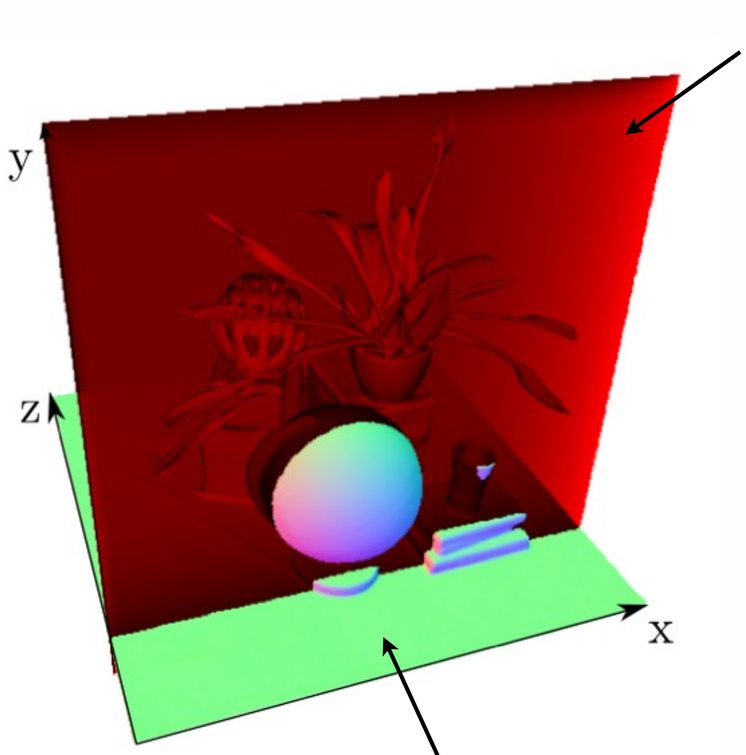
Rigid Scanning - Accumulation

Volumetric accumulation

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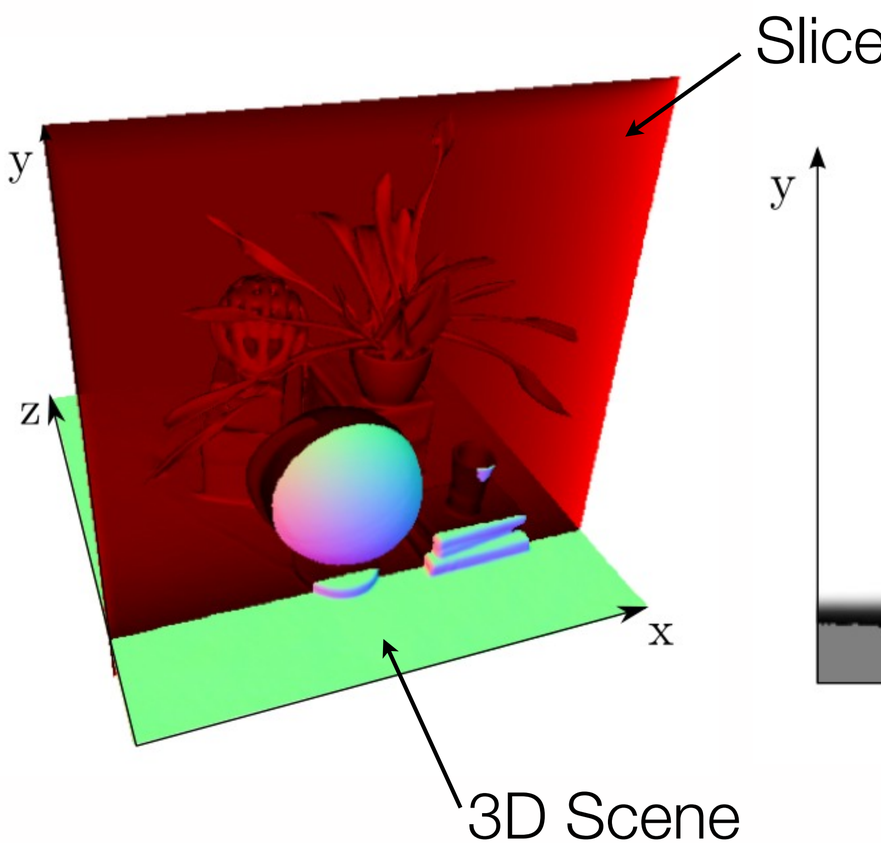
Volumetric accumulation

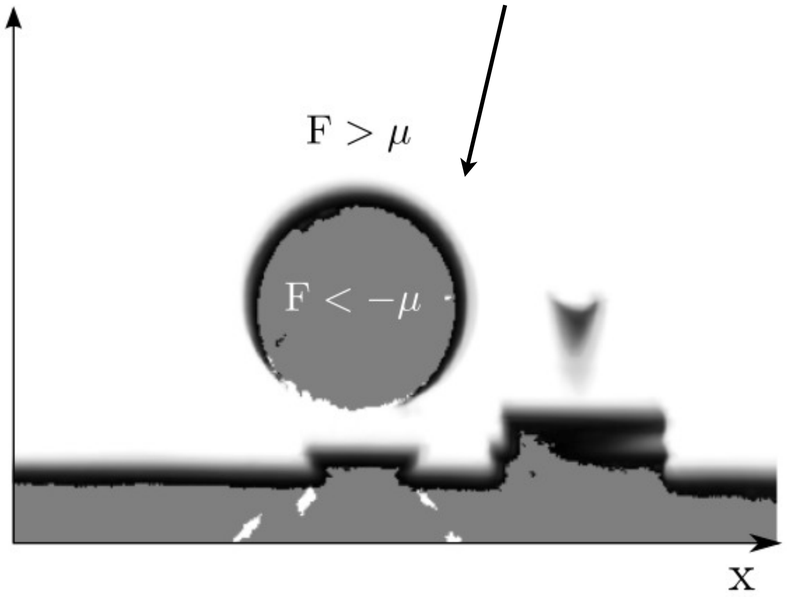


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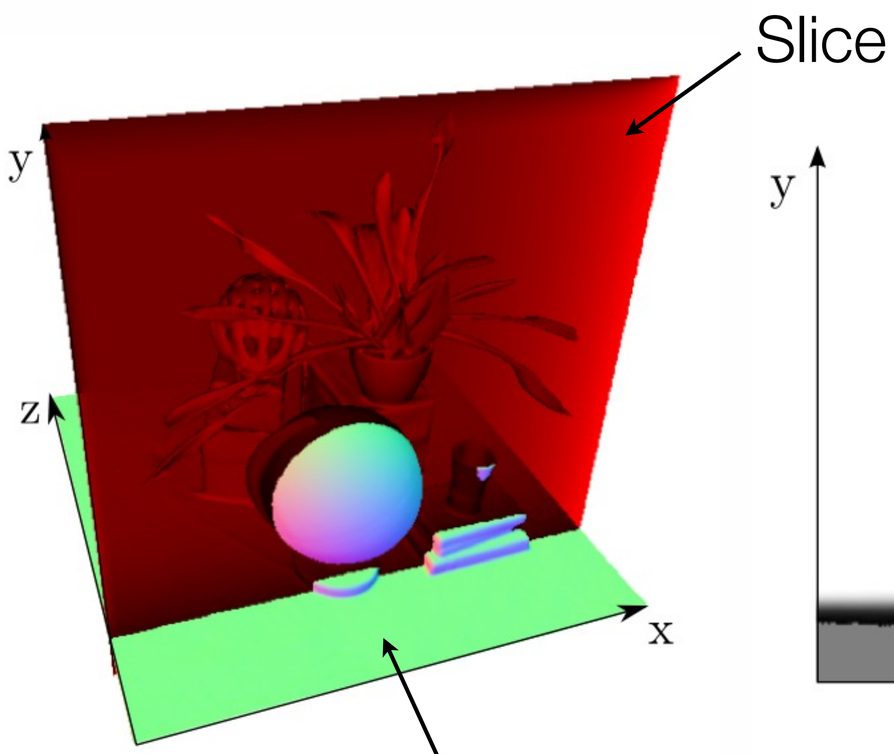


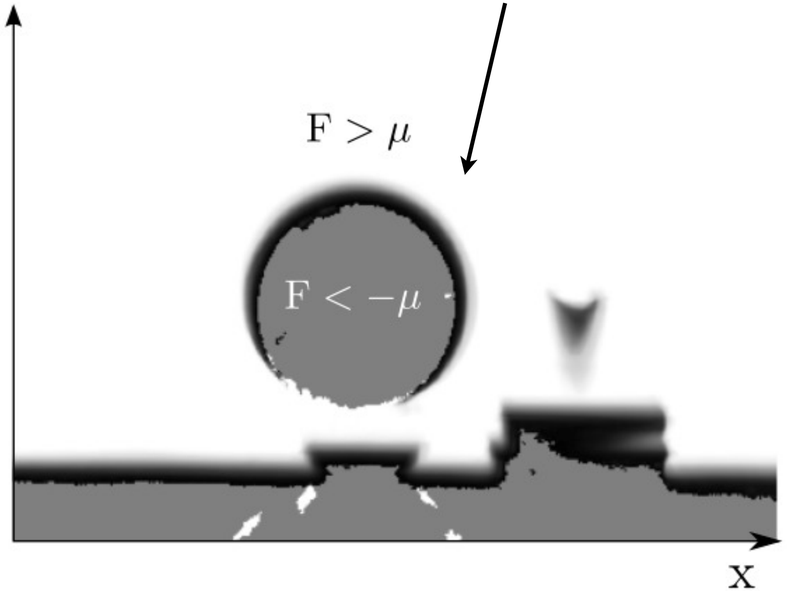


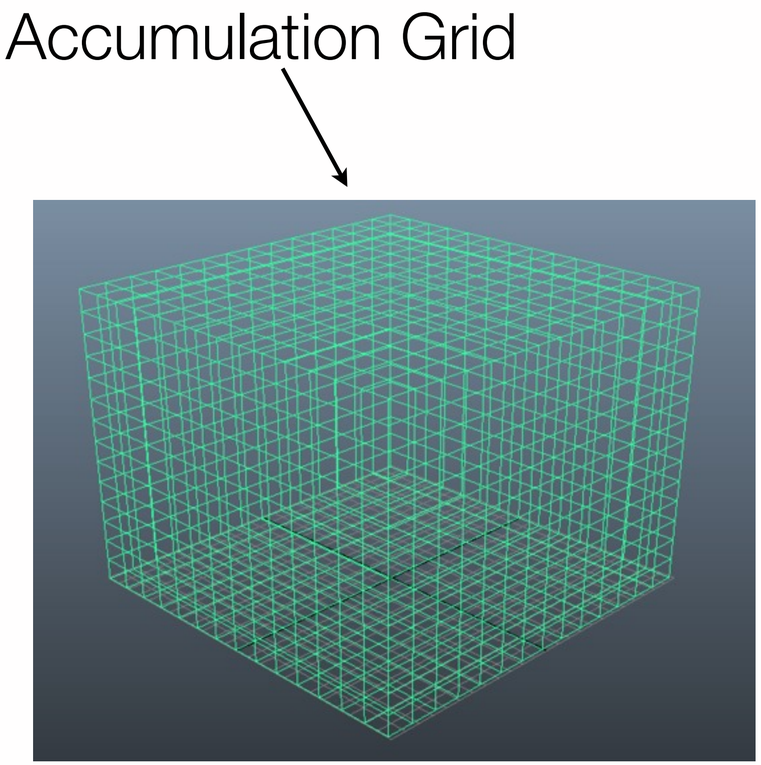
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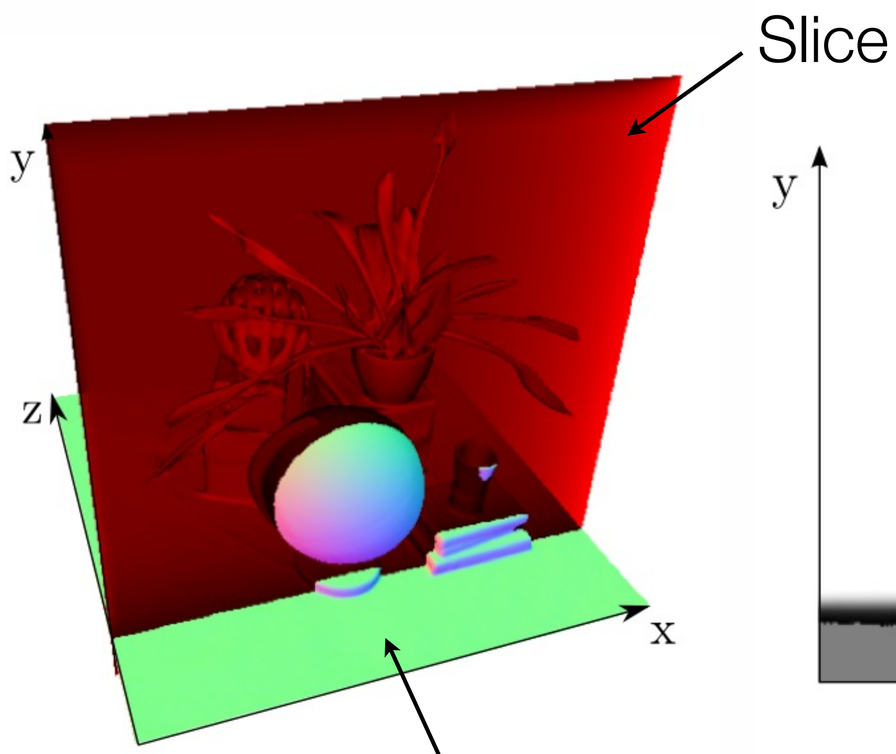


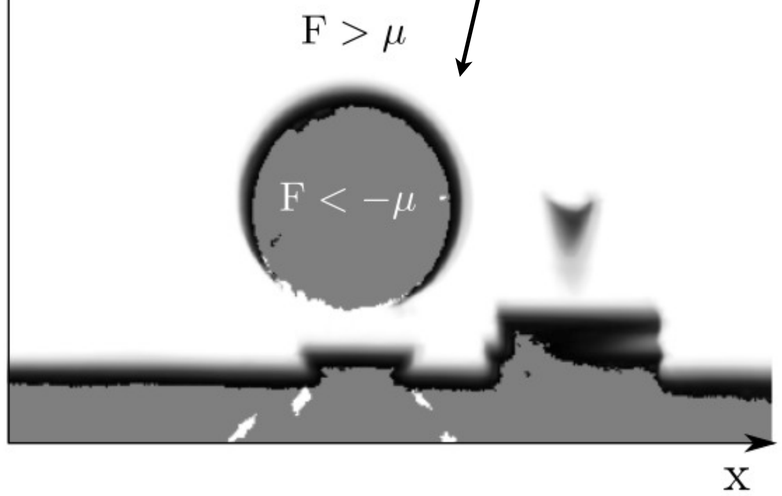


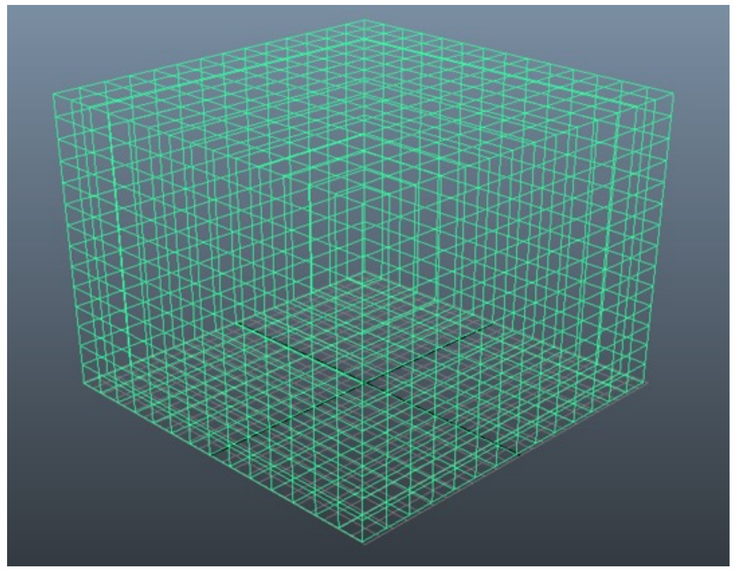
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Rigid Scanning - Accumulation

Volumetric accumulation







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DEMO

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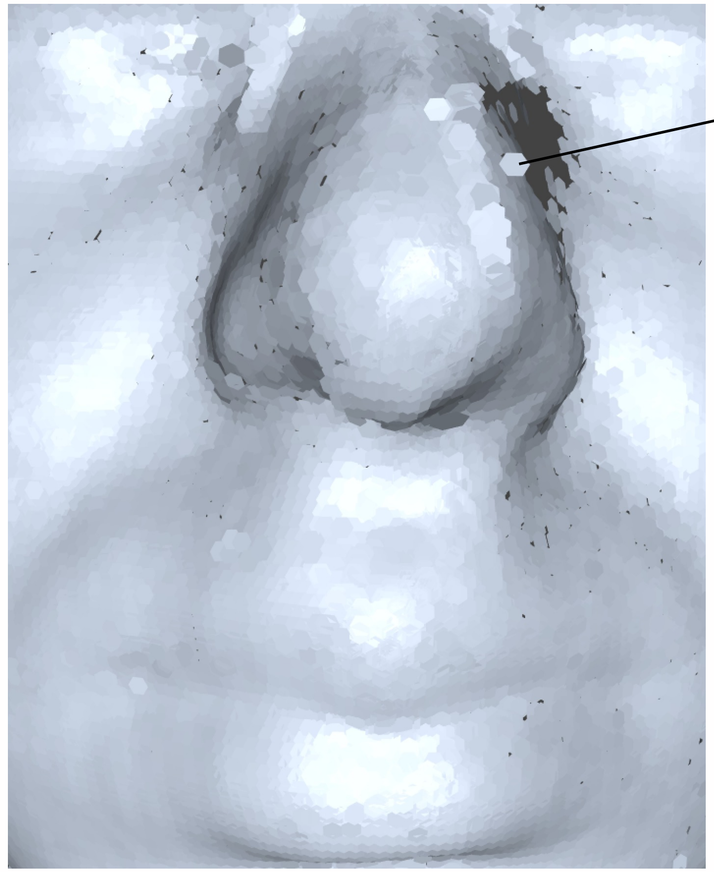
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Rigid Scanning - Accumulation

Surface accumulation





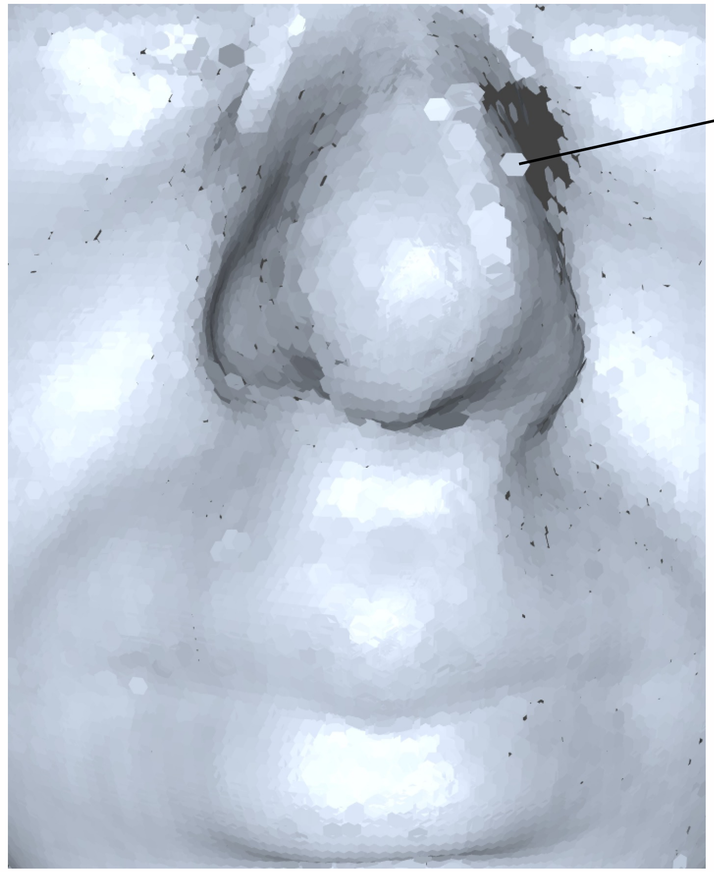


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Rigid Scanning - Accumulation

Surface accumulation

Surfel update







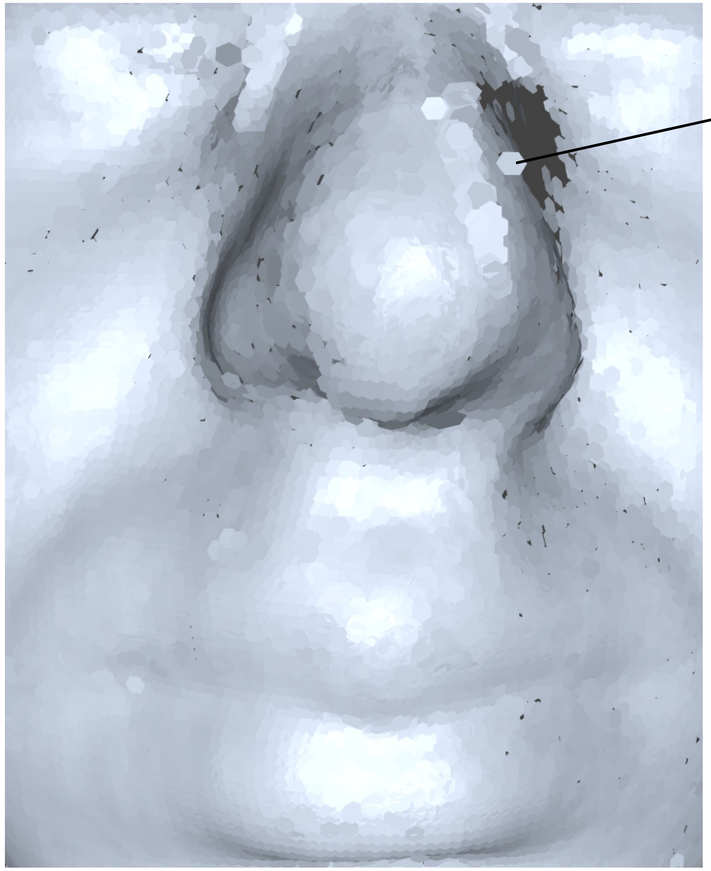
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Rigid Scanning - Accumulation

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Surfel update

Surfel addition







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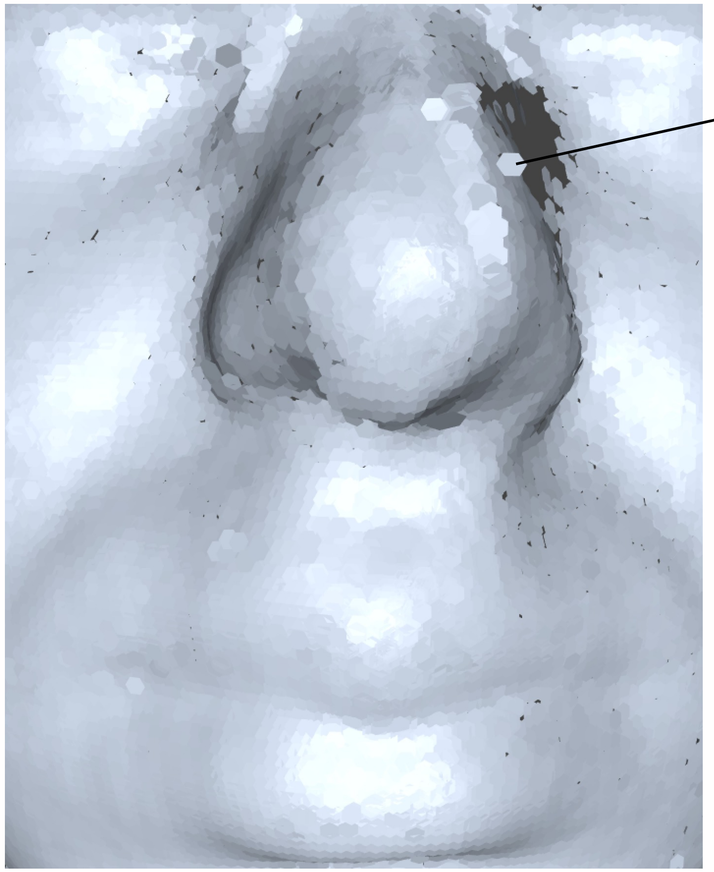
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Surface accumulation

Surfel update

Surfel addition

Surfel removal







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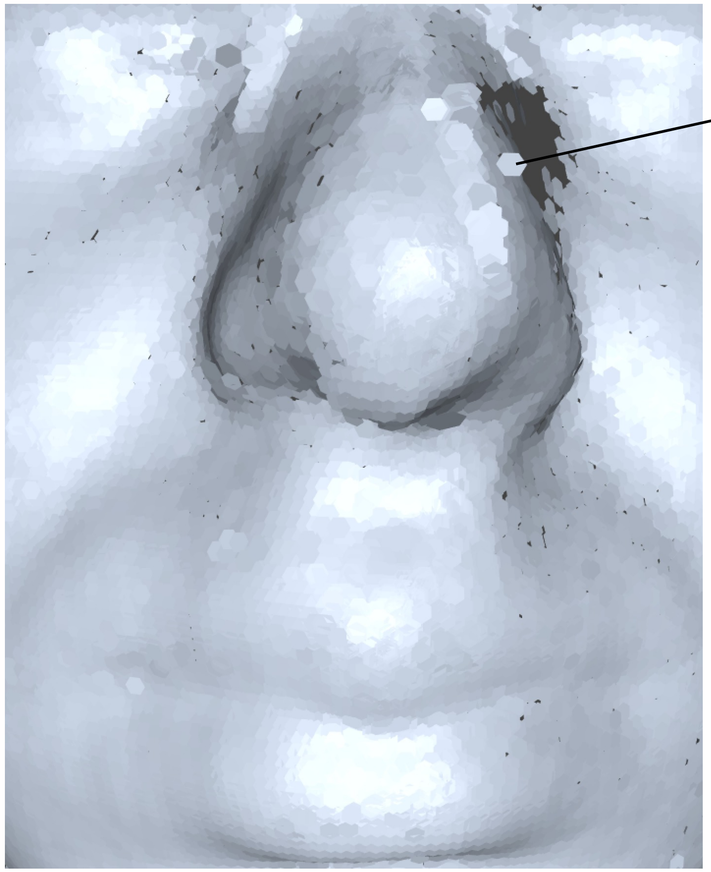
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DEMO



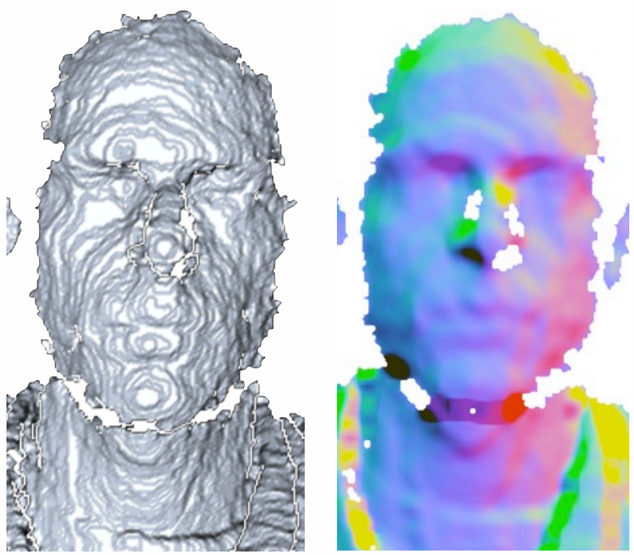
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Rigid Scanning - Recap

Rigid Scanning - Recap

Preprocessing

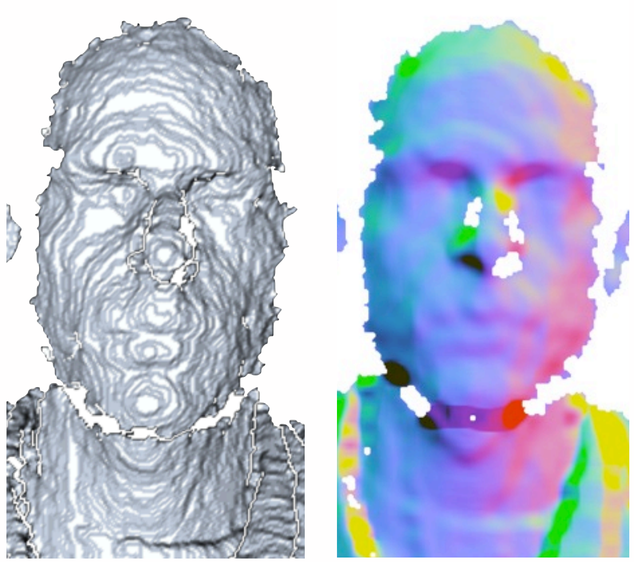
$$\mathbf{x}_{ij} = \mathbf{K}^{-1} \begin{bmatrix} i \cdot d(i,j) \\ j \cdot d(i,j) \\ d(i,j) \end{bmatrix}$$



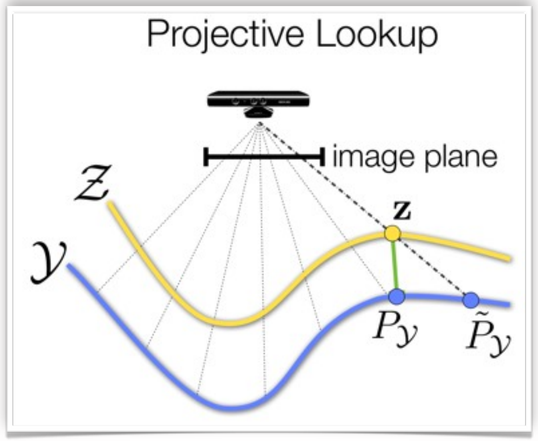
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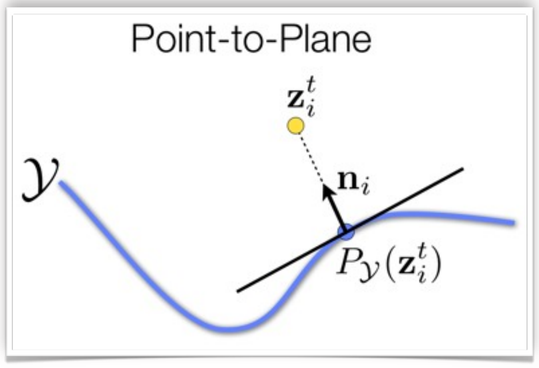
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Registration

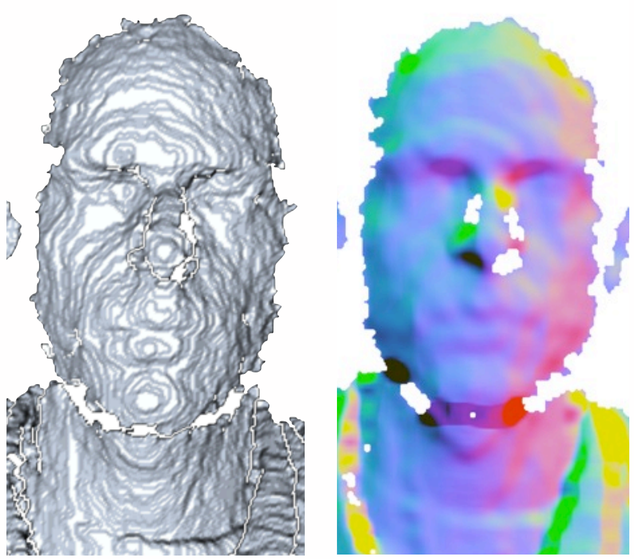




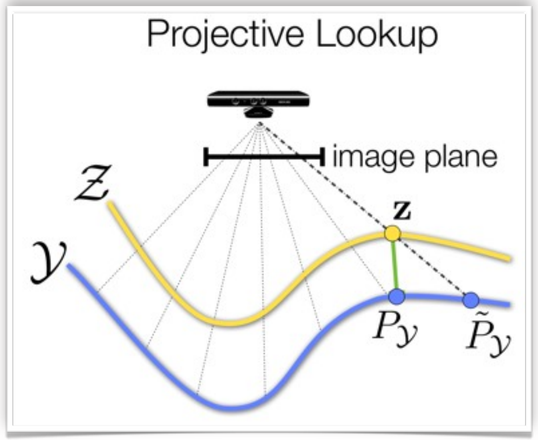
Rigid Scanning - Recap

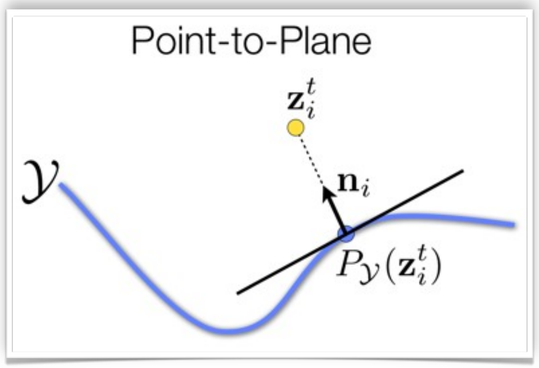
Preprocessing

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Registration





Accumulation (reconstruction)

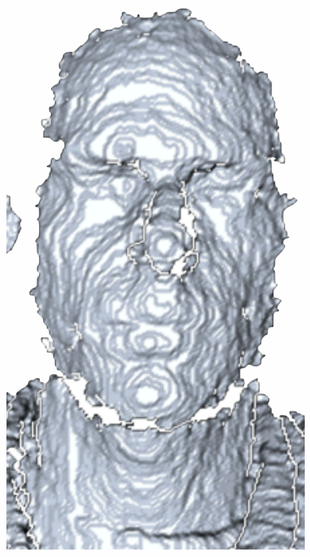


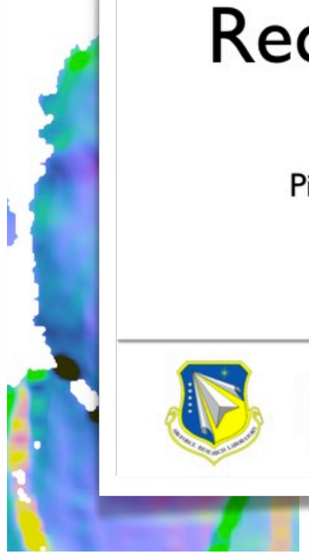


Rigid Scanning - Recap

Preprocessing

$$\mathbf{x}_{ij} = \mathbf{K}^{-1} \begin{bmatrix} i \cdot d \\ j \cdot d \\ d(i,j) \end{bmatrix}$$





Registration

EUROGRAPHICS 2014
Strasbourg, France

State of the Art in Surface Reconstruction from Point Clouds

Matthew Berger Andrea Tagliasacchi Lee M. Seversky
Pierre Alliez Joshua A. Levine Andrei Sharf Claudio T. Silva

Accumulation (reconstruction)



Overview

Introduction

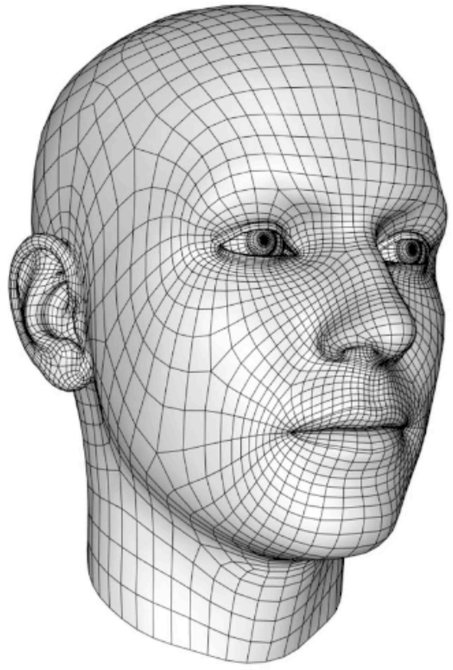
- Registration
  - 3D Geometry
  - 2D Images
  - Combined 2D/3D
  - Robust Registration
  - Q&A
- Applications
  - Rigid Scanning
  - 
    Non-rigid Modeling
  - Realtime Face Tracking

Conclusions

Q&A

Non-Rigid Modeling

Non-Rigid Modeling





Non-Rigid Modeling

$$\underset{Z,\{\mathbf{R}_i\}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

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matching term

Non-Rigid Modeling

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matching term

non-rigid prior

Non-Rigid Modeling

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- How to set the weight for the non-rigid energies?

Non-Rigid Modeling

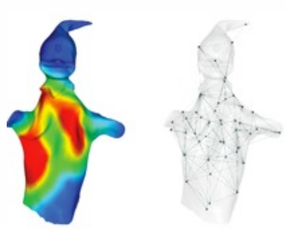
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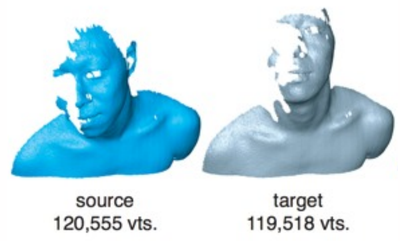
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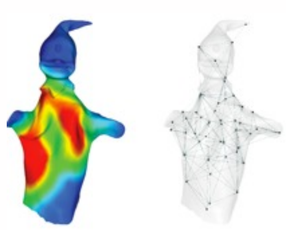


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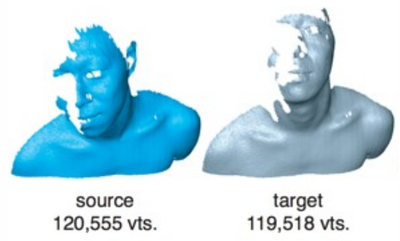
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DEMO



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Non-Rigid Modeling

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Non-Rigid Modeling

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Non-Rigid Modeling

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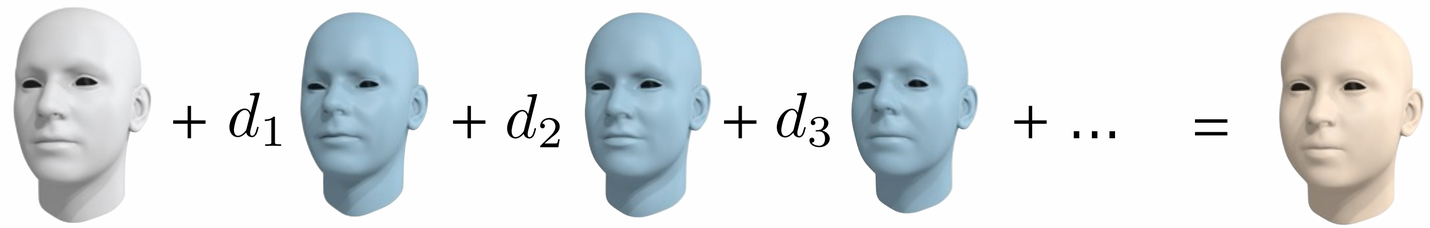
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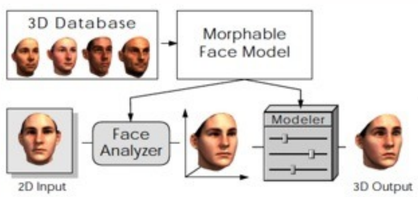
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Non-Rigid Modeling

- PCA model

$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$





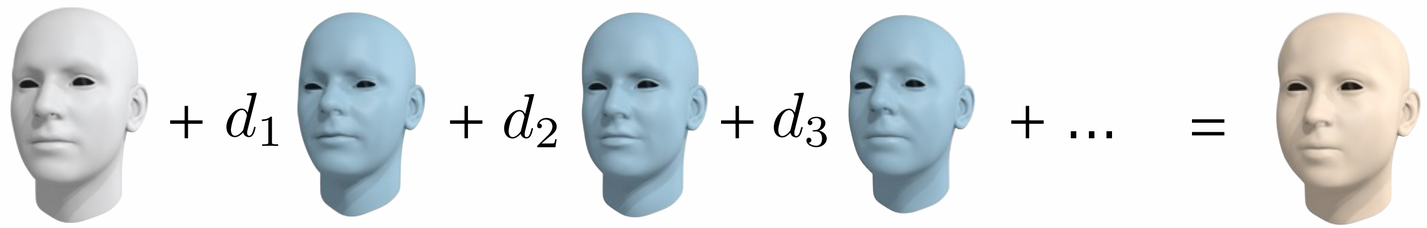
Blanz, Vetter: **A Morphable Model for the Synthesis of 3D Faces**
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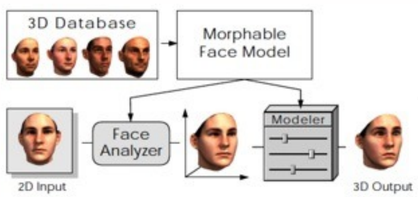
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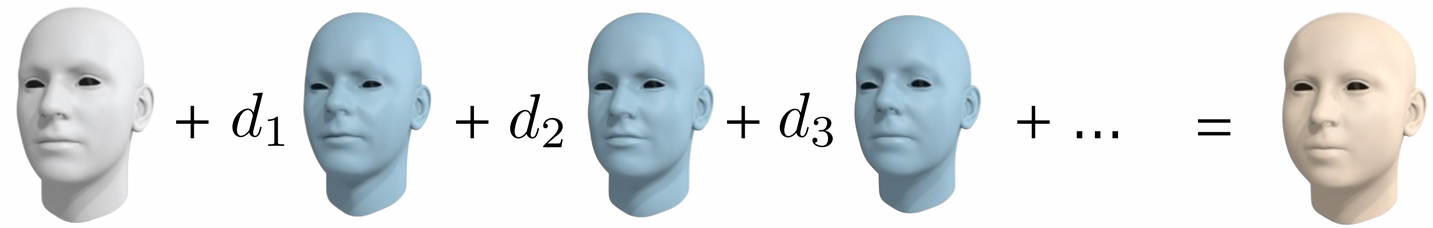
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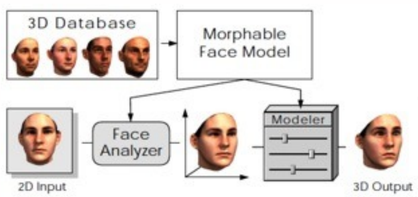
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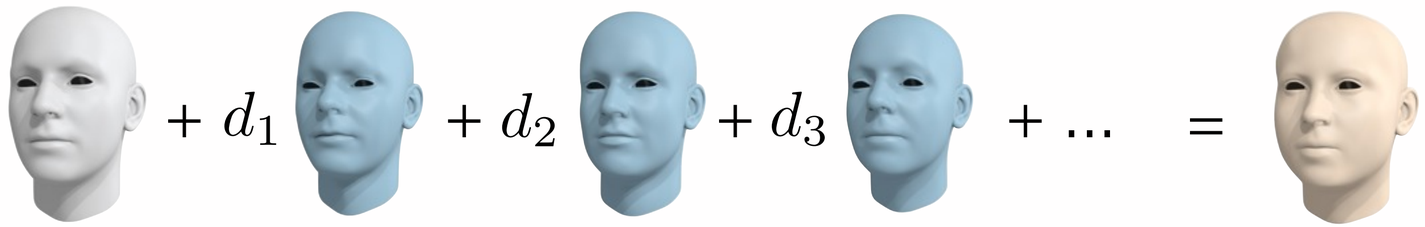
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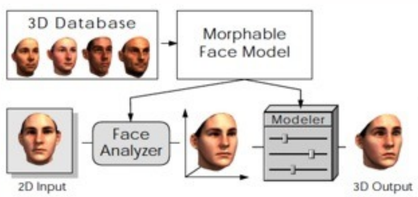
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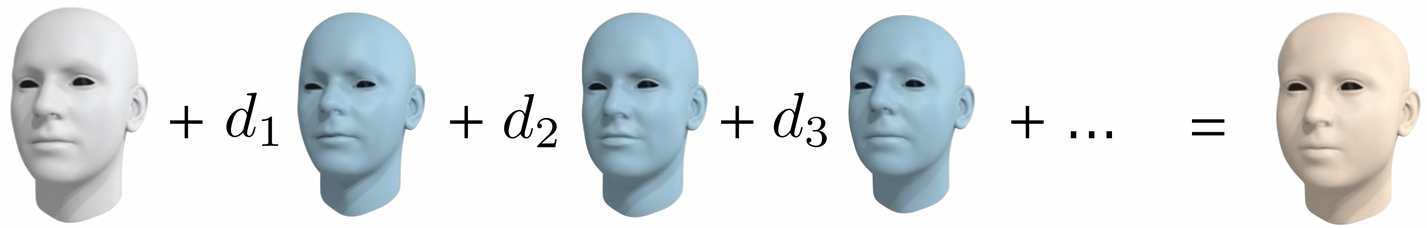
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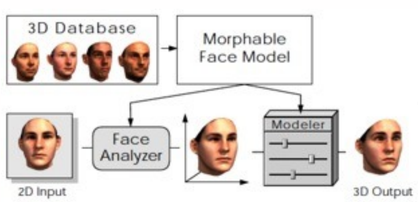
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$$|\mathbf{d}| << |\mathcal{V}|$$

50 7400

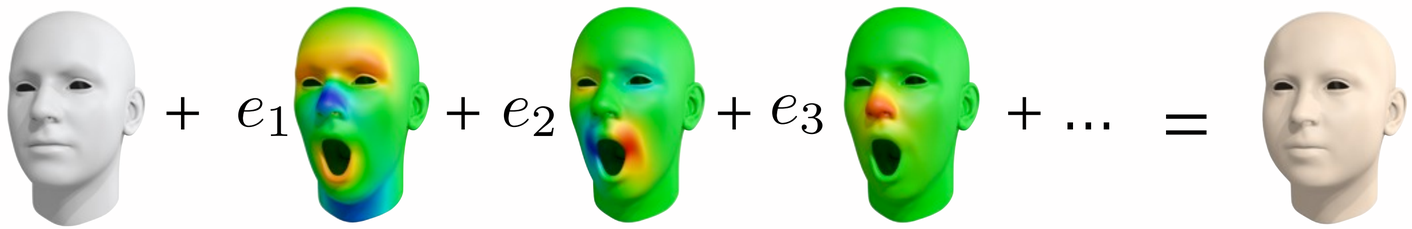


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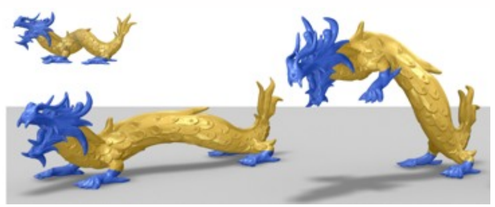
Non-Rigid Modeling

- Modal Analysis (subspace optimization)
  - Assume smooth deformation (no high frequency)
  - Eigenmodes of the Hessian of the deformation energy

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Hildebrandt, Schulz, Tycowicz, Polthier:
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ACM SIGGRAPH 2012

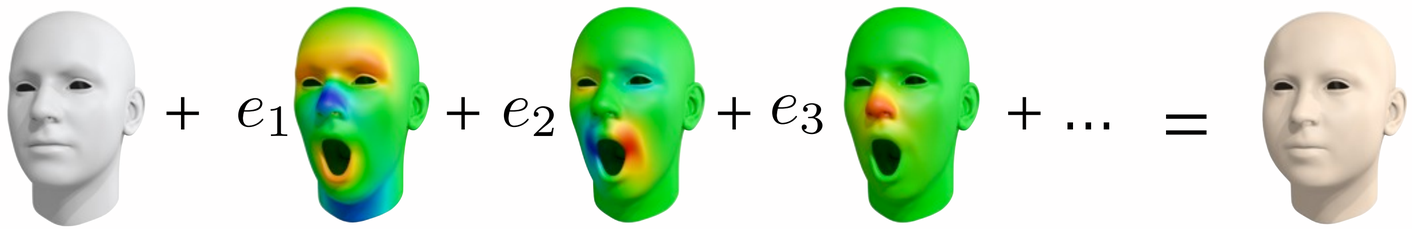


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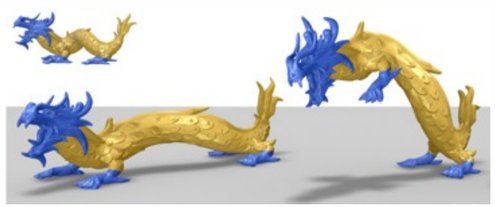
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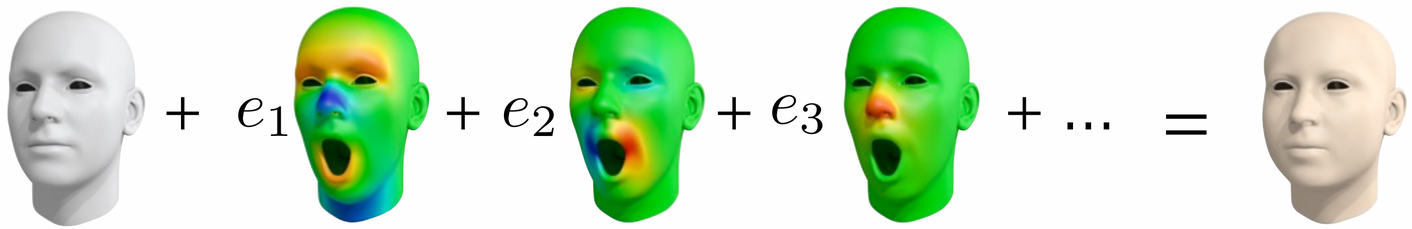
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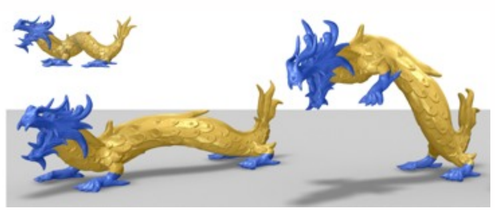
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basis

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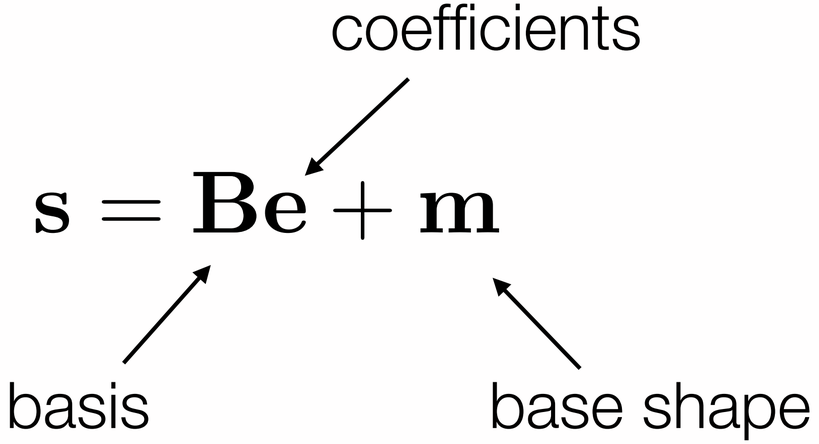


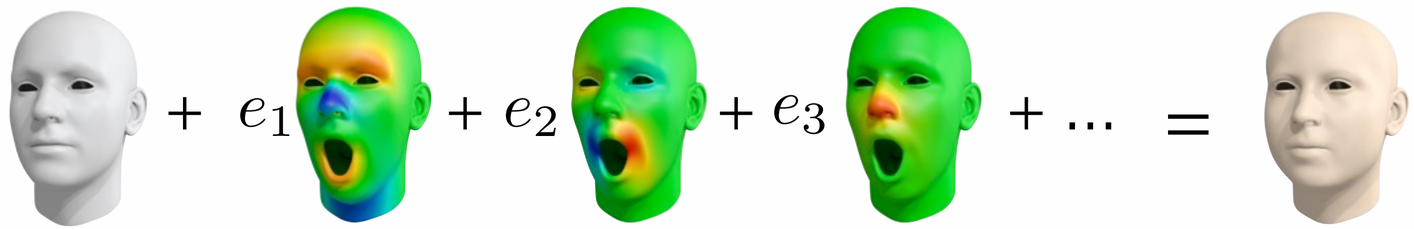
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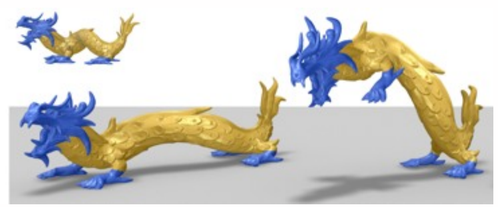
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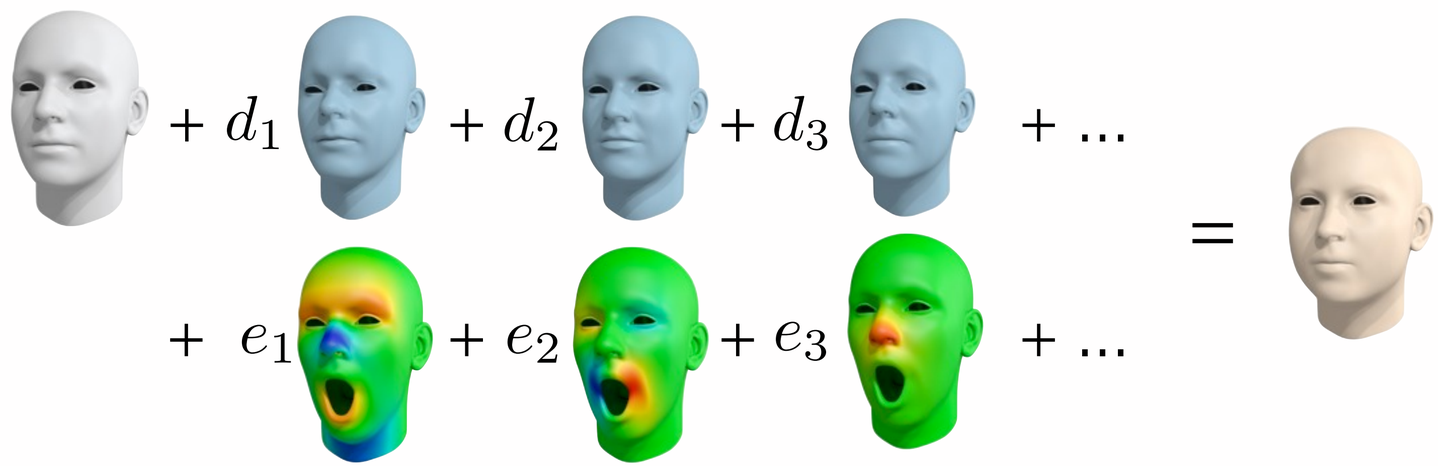


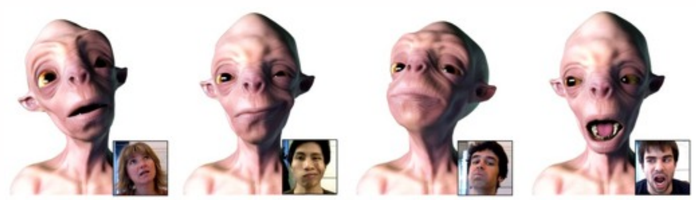
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Non-Rigid Modeling

- Combining PCA with Modal Analysis



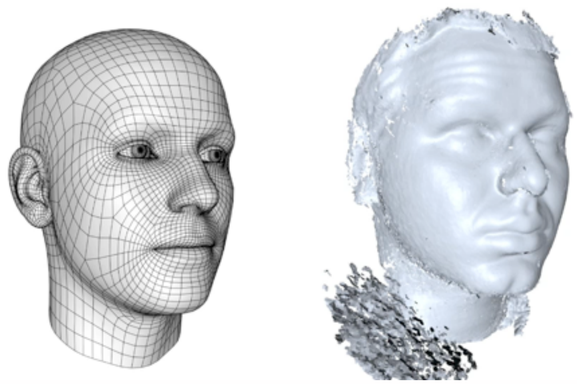


Bouaziz, Wang, Pauly:
Online Modeling For Realtime Facial Animation,
ACM SIGGRAPH 2013

Non-Rigid Modeling - Recap

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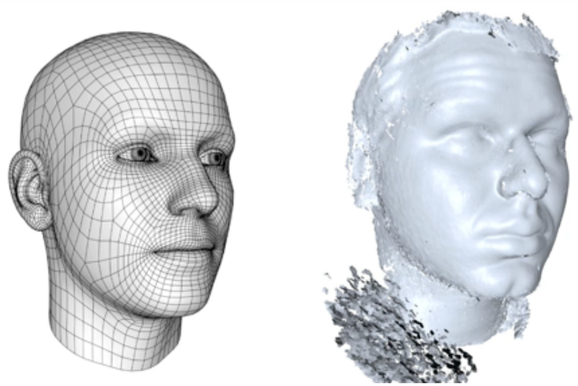
Registration



$$\arg\min_{Z,\{\mathbf{R}_i\}} w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$
$$+ w_2 \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

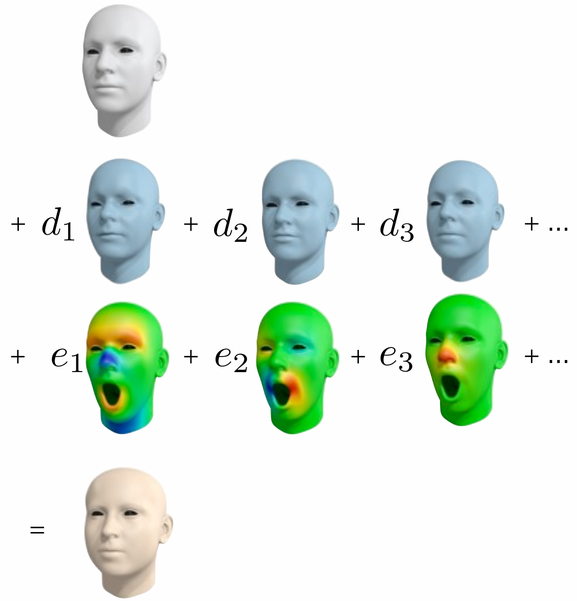
Non-Rigid Modeling - Recap

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Advanced Prior



1) Data-Driven: Statistical model (PCA)
2) Physical-Based: Modal analysis

Overview

Introduction

- Registration
  - 3D Geometry
  - 2D Images
  - Combined 2D/3D
  - Robust Registration
  - Q&A
- Applications
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  - Non-rigid Modeling



  - Realtime Face Tracking

Conclusions

Q&A

Face Tracking

- Blendshape model

Face Tracking

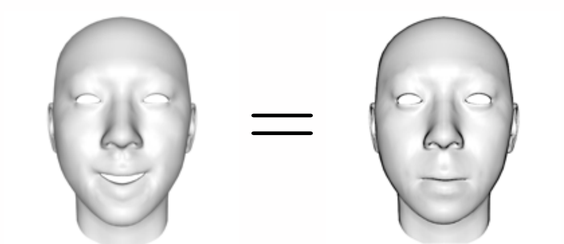
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happy smile

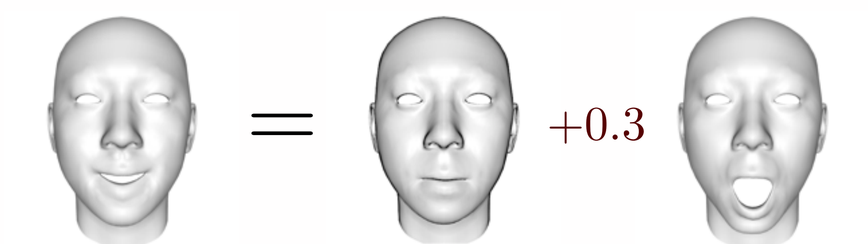
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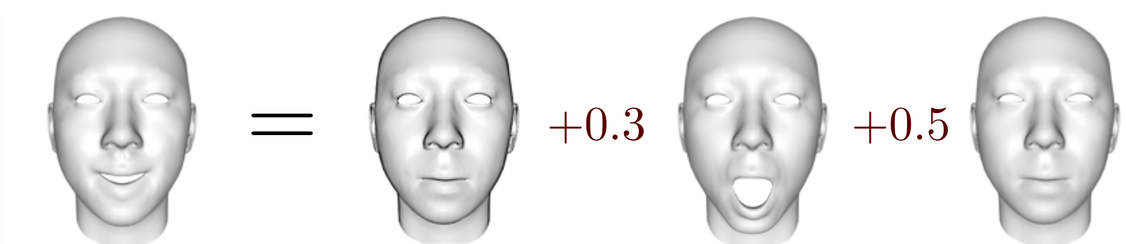
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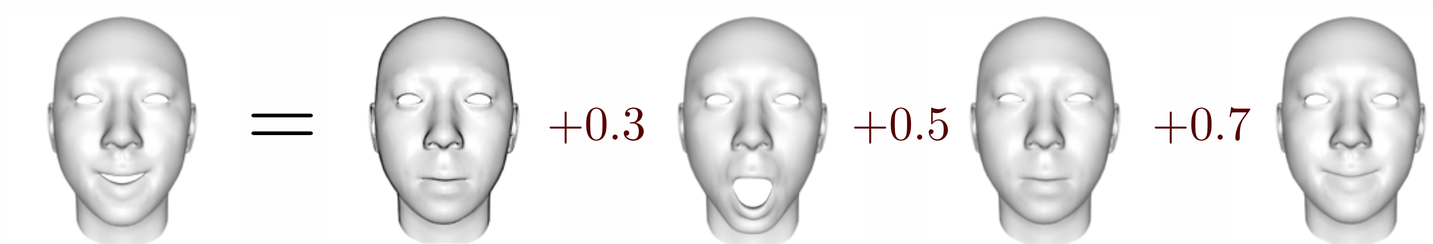
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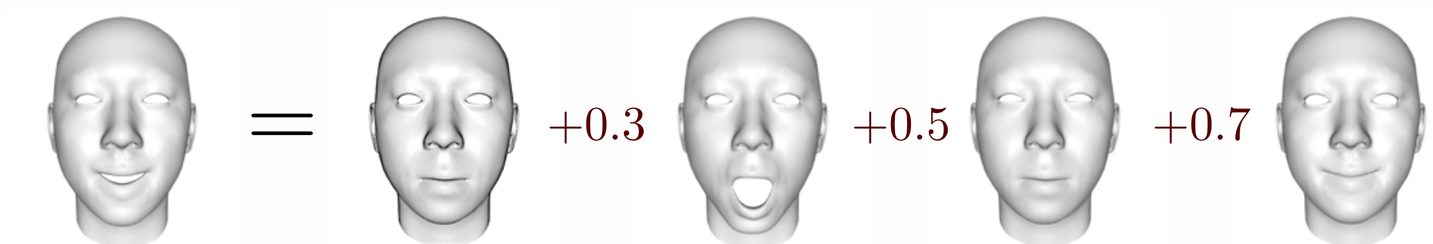
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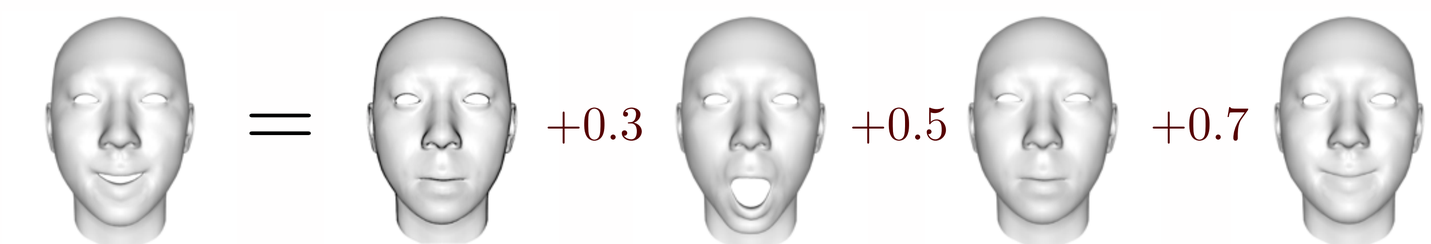
- Blendshape model



$$\mathbf{e} = \mathbf{B}\mathbf{d} + \mathbf{b}$$

Face Tracking

- Blendshape model

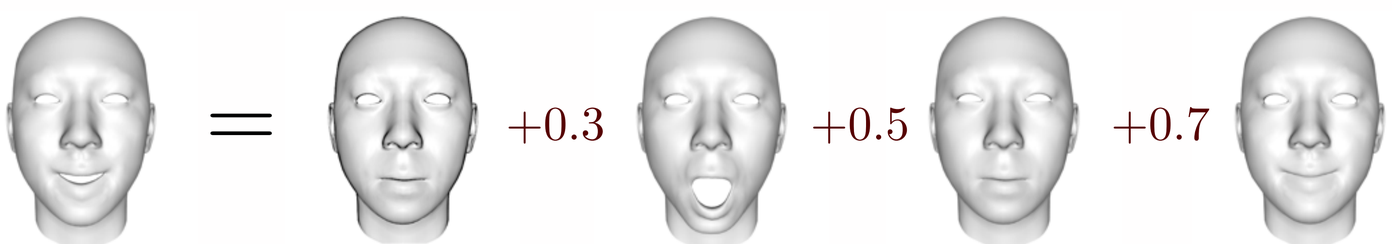


$$\mathbf{e} = \mathbf{B}\mathbf{d} + \mathbf{b}$$

novel expression

Face Tracking

- Blendshape model



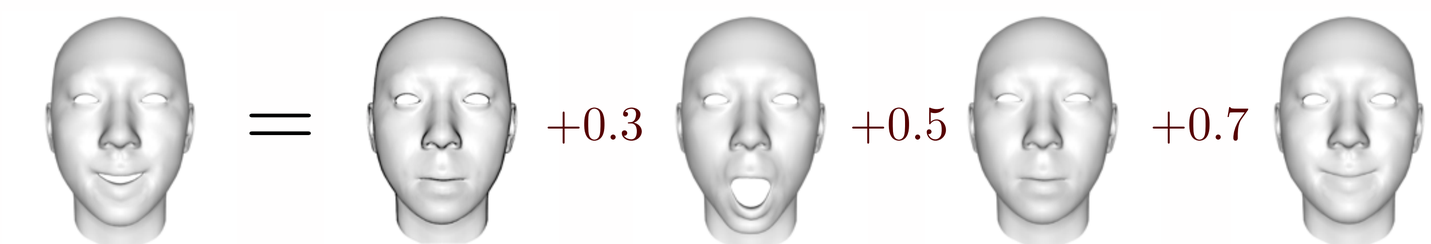
$$\mathbf{e} = \mathbf{B}\mathbf{d} + \mathbf{b}$$

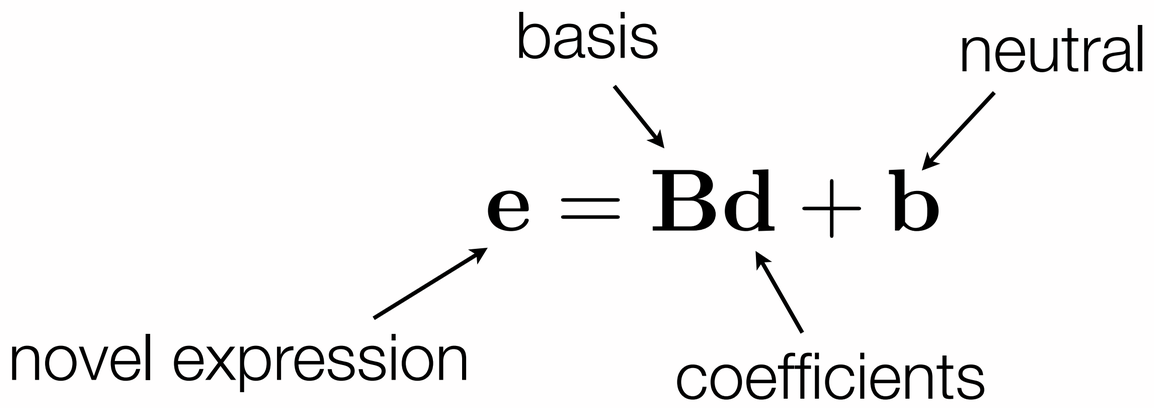
neutral

novel expression

Face Tracking

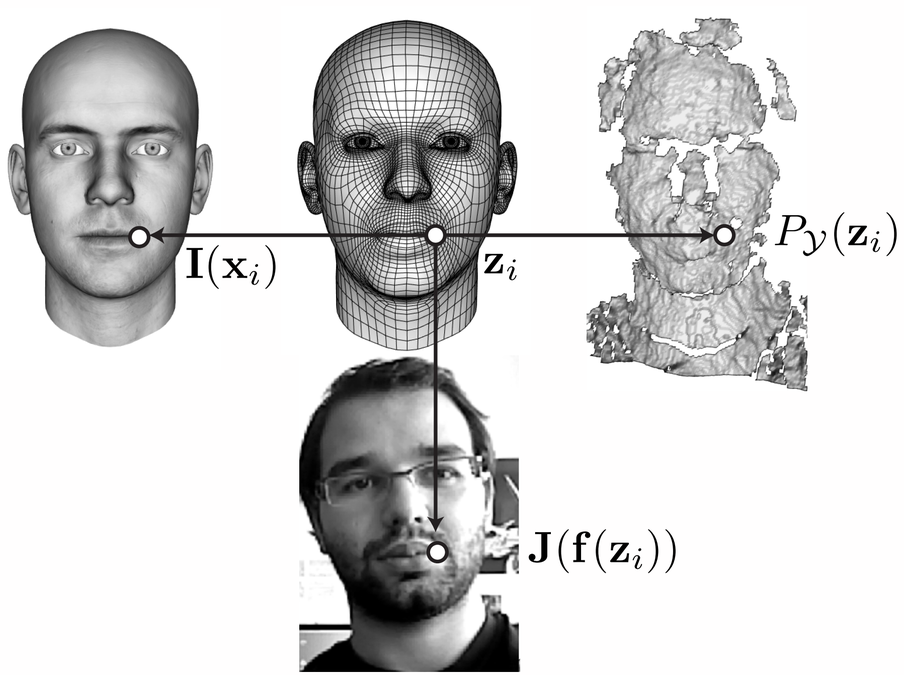
- Blendshape model





Face Tracking

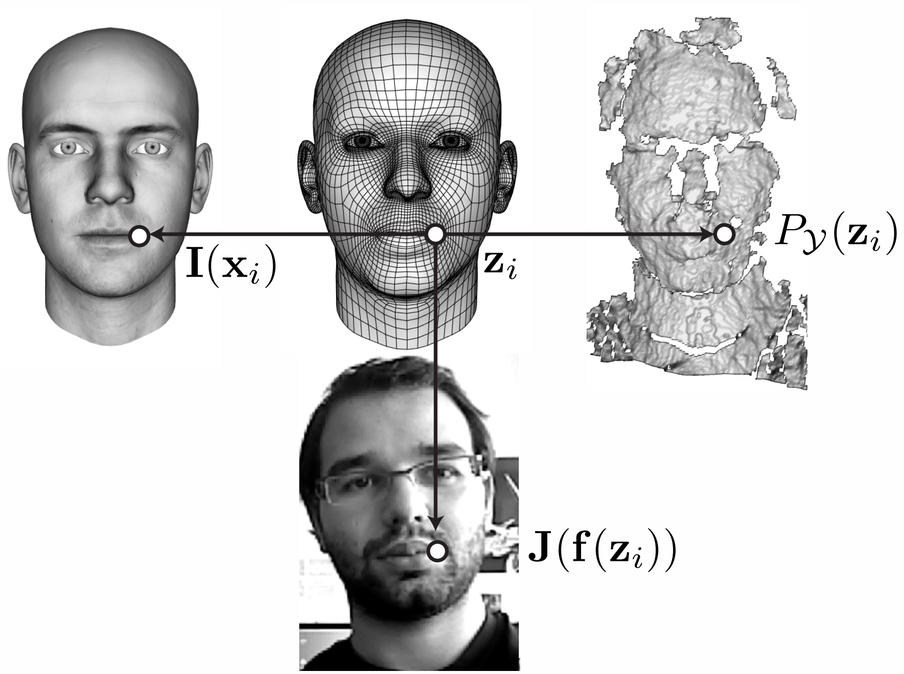
$$\underset{Z,\mathbf{d},\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i\mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$



Face Tracking

$$\underset{Z,\mathbf{d},\mathbf{R},\mathbf{t}}{\arg\min}\, w_1 \sum_{i=1}^{n} \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i\mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$

3D matching term

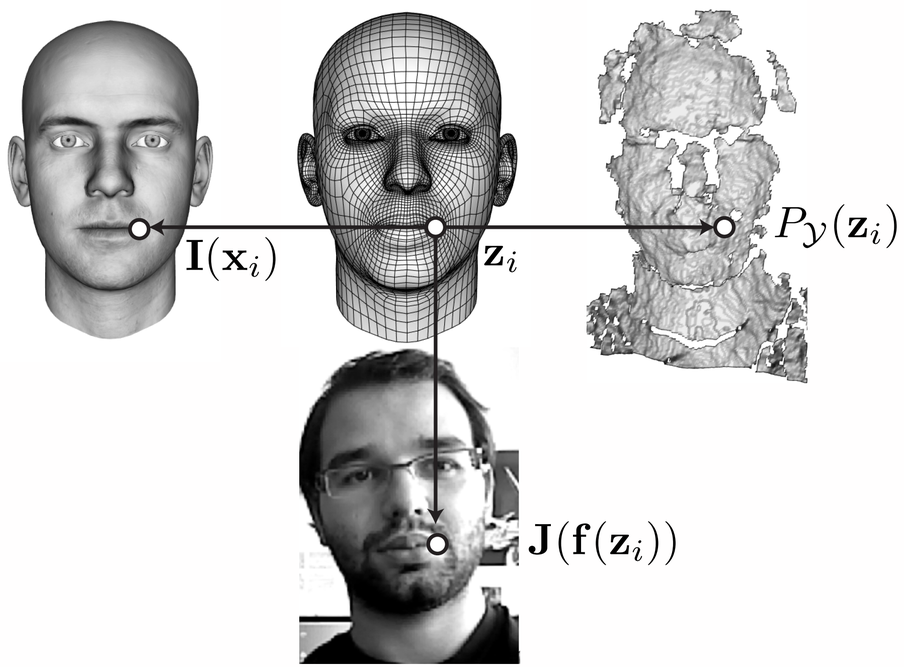


Face Tracking

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3D matching term

2D matching term



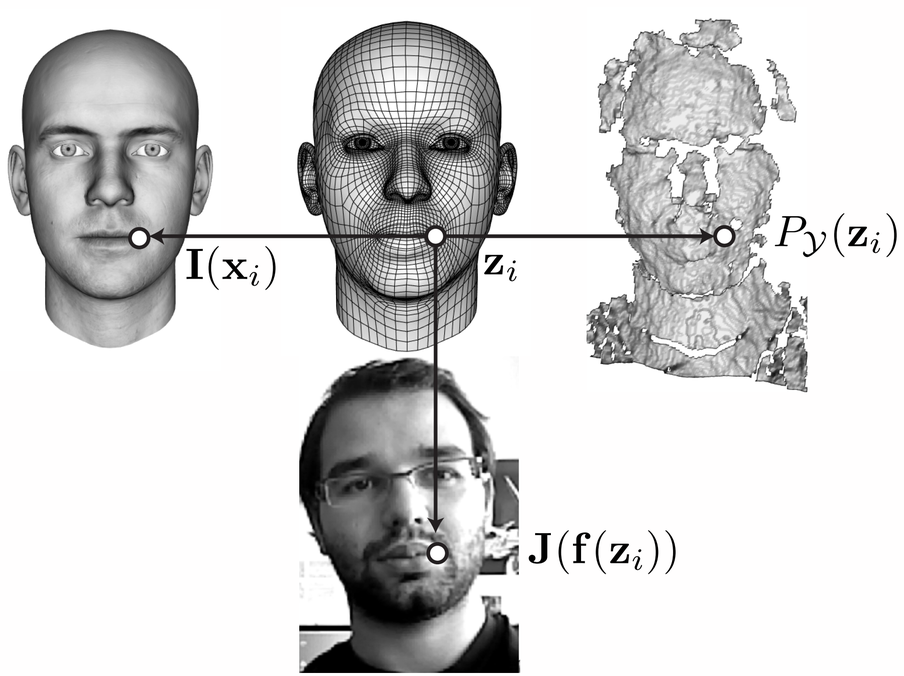
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3D matching term

2D matching term

blendshapes-rigid motion



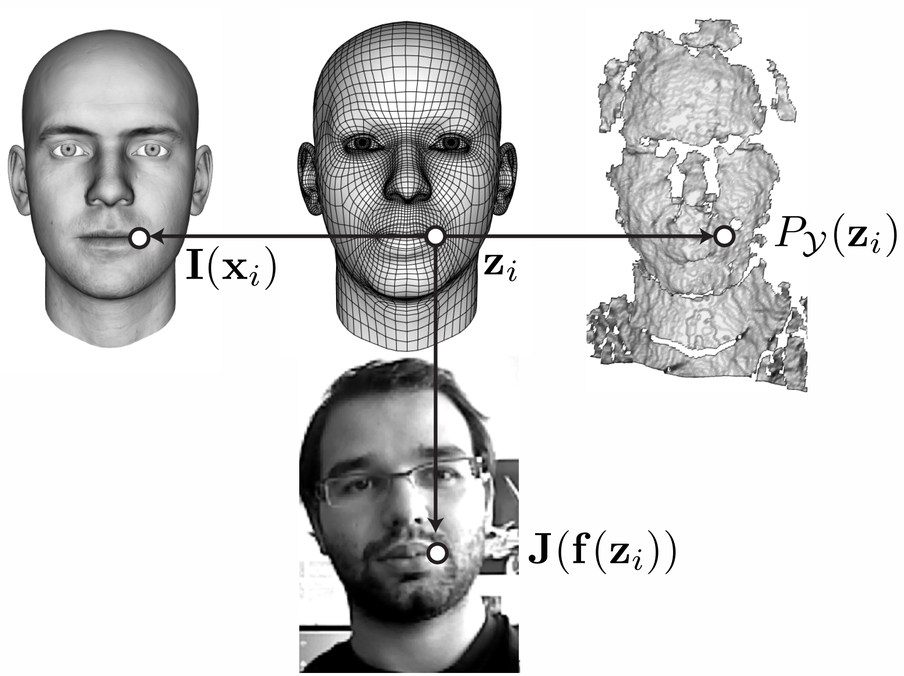
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DEMO

Face Tracking

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3D matching term

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Face Tracking

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3D matching term

2D matching term

blendshapes/rigid motion

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blendshapes/rigid motion

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Iterate until convergence

Face Tracking

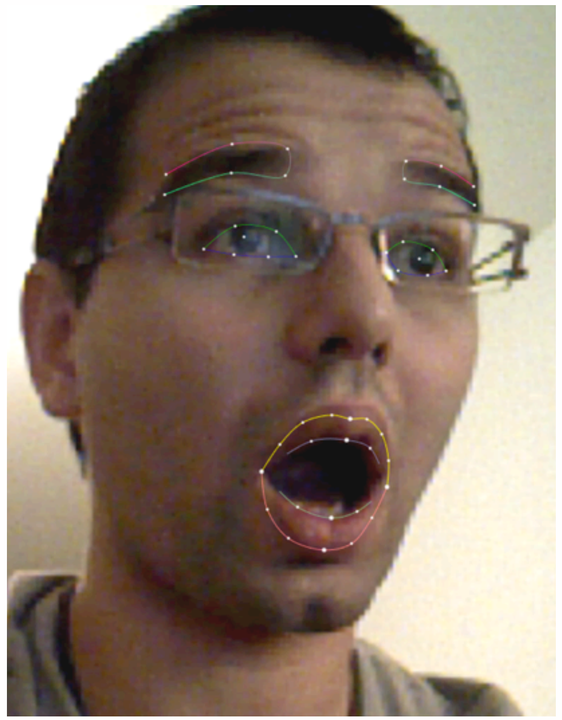
- Until now - registration using local optimization

Face Tracking

- Until now - registration using local optimization
- Other possibility - alignment using descriptors and features

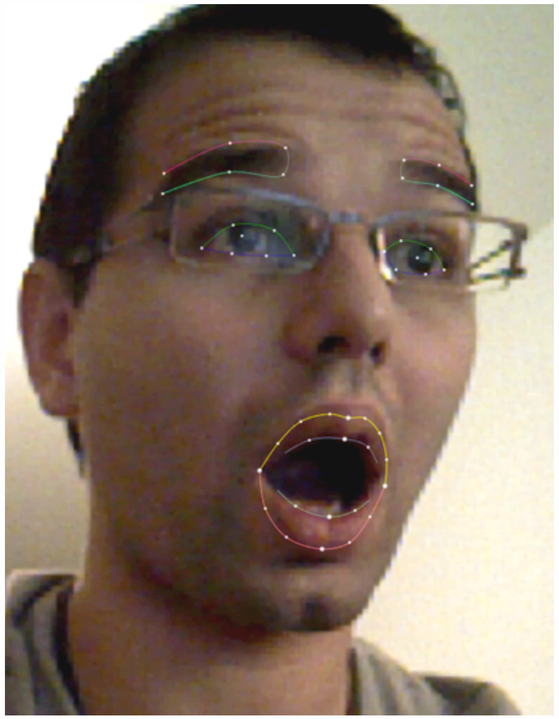
Face Tracking

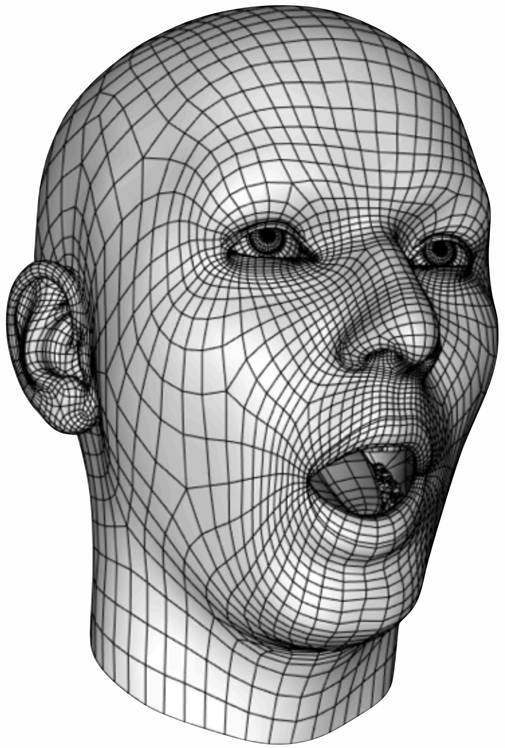
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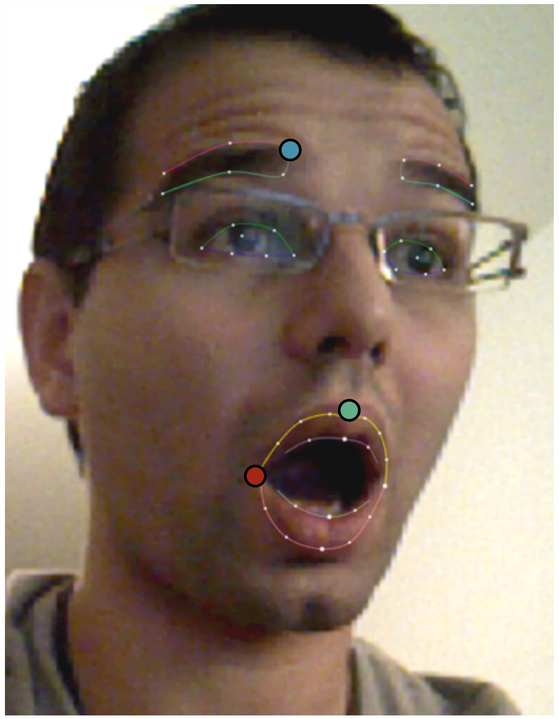
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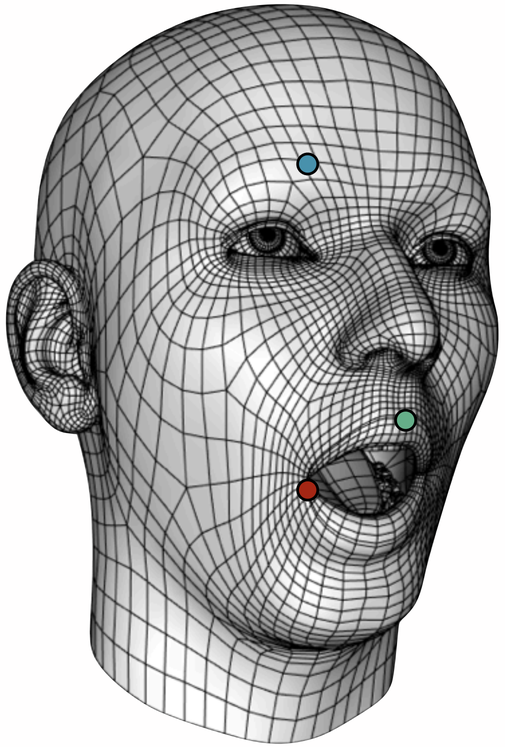




Face Tracking

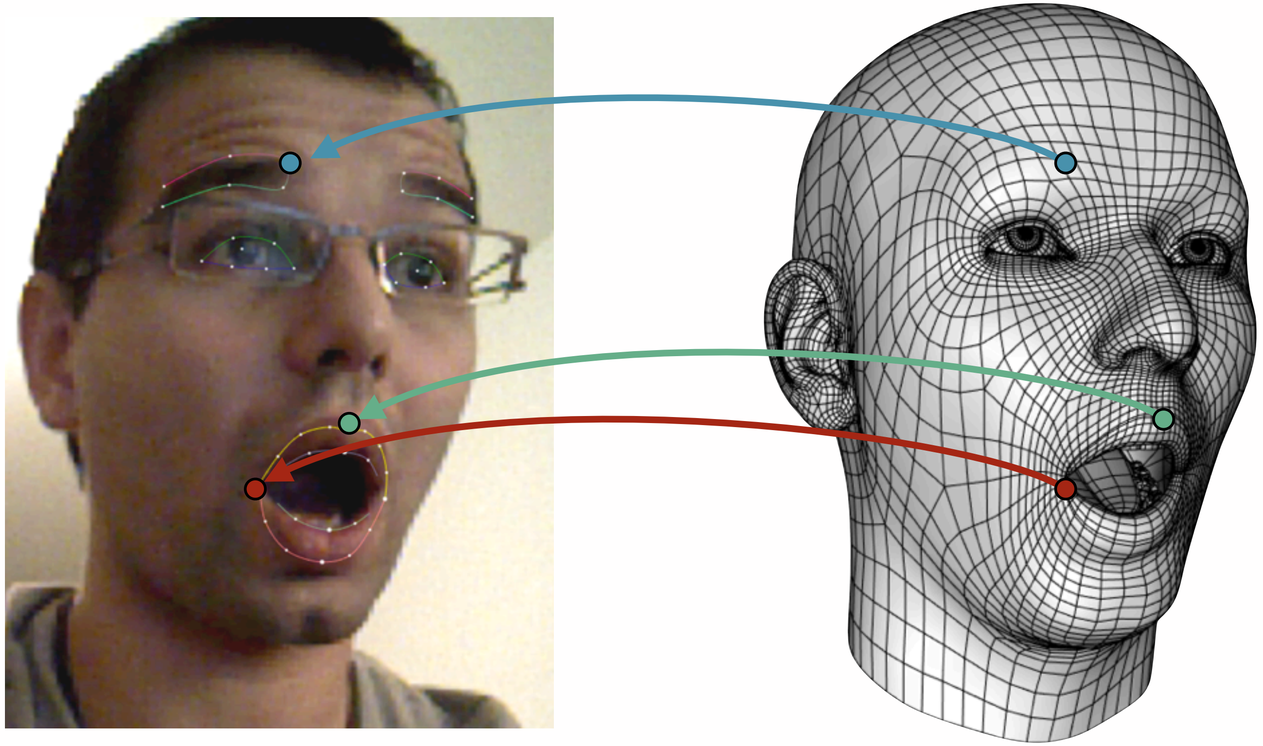
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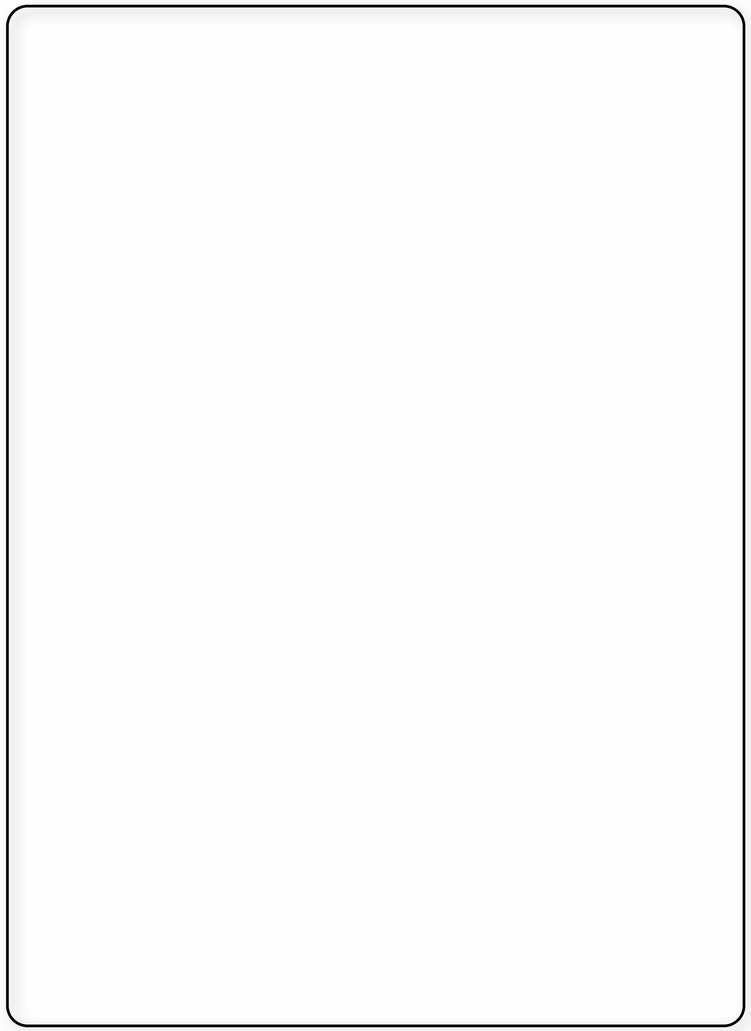


Face Tracking

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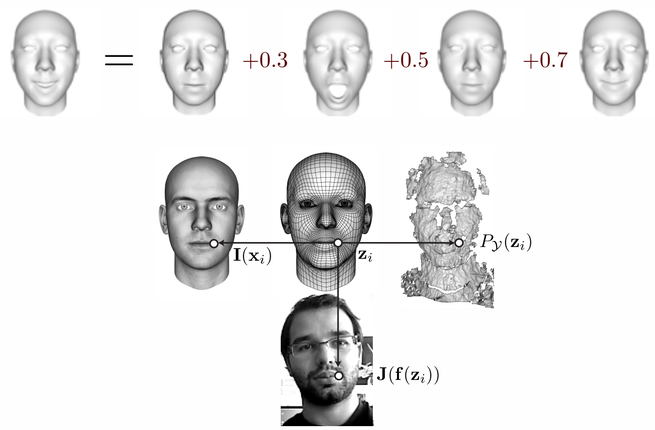


Face Tracking - Recap



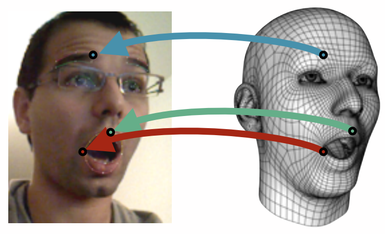
Face Tracking - Recap

Registration



Step 1: optimize rigid motion

Step 2: optimize coefficients



Overview

Introduction

- Registration
  - 3D Geometry
  - 2D Images
  - Combined 2D/3D
  - Robust Registration
  - Q&A
- Applications
  - Rigid Scanning
  - Non-rigid Modeling
  - Realtime Face Tracking



Conclusions

Q&A

Summary

- Registration as energy minimization

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  - general framework for integrating image and geometry registration

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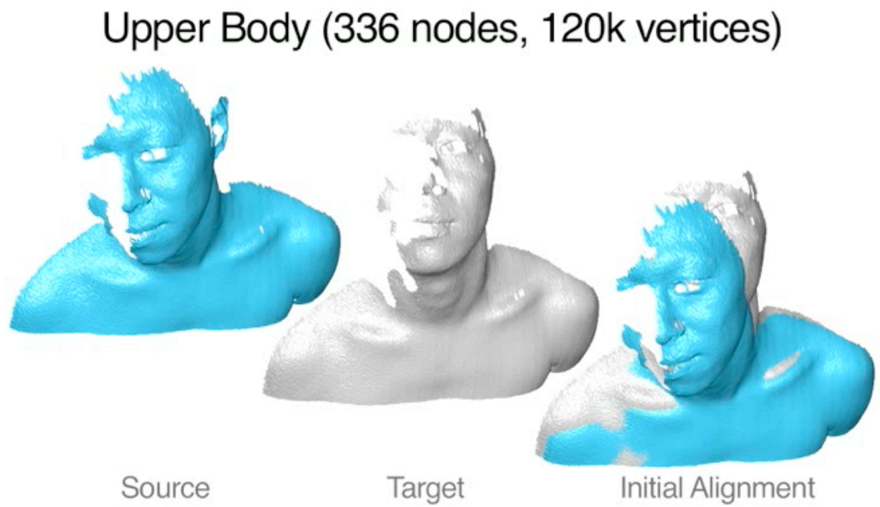
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Li, Sumner, Pauly, **Global Correspondence Optimization for Non-Rigid Registration of Depth Scans**, *SGP 2008*

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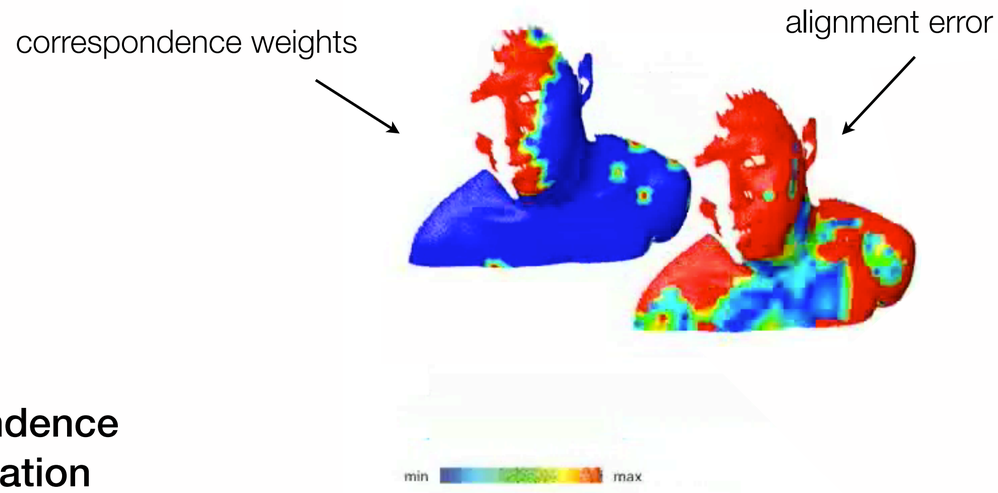
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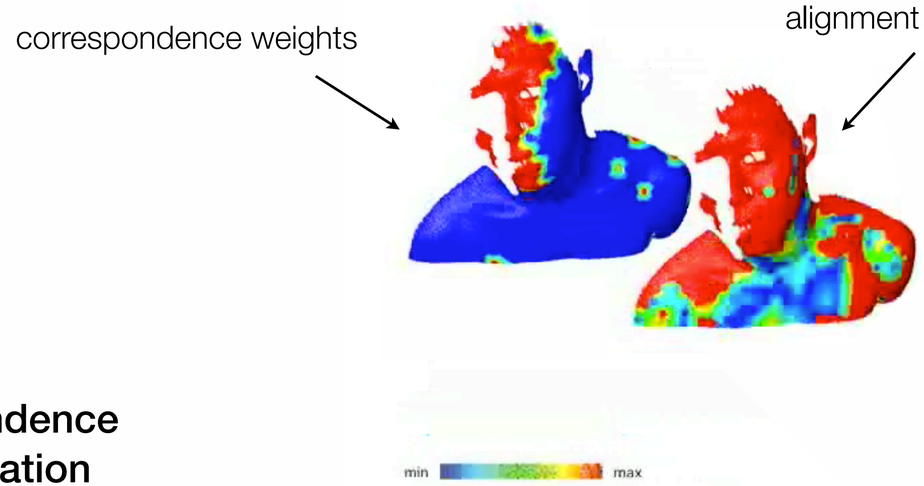
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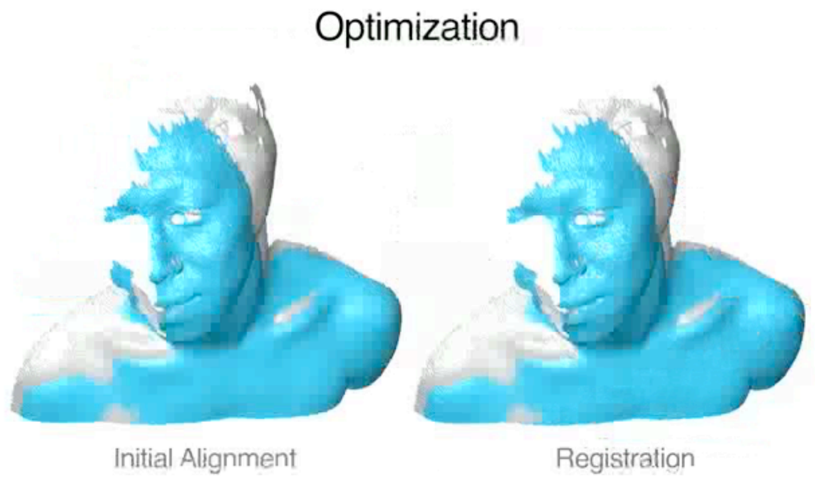


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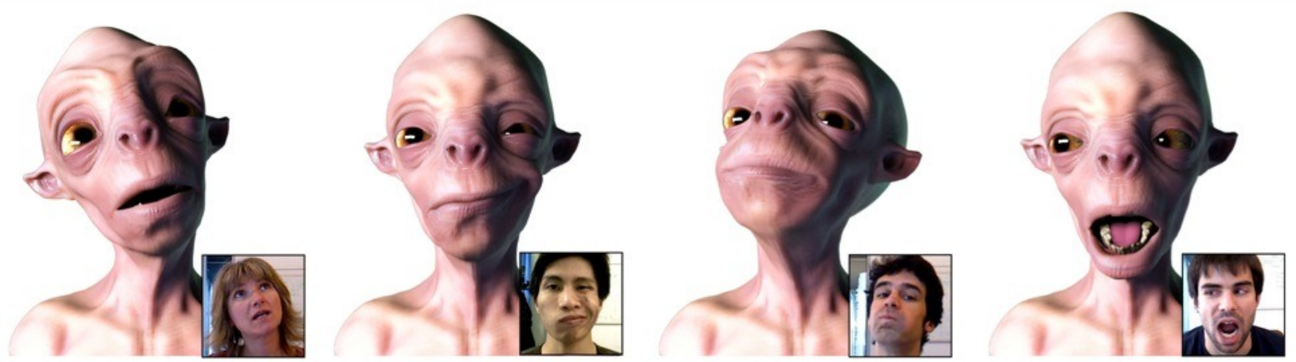
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Bouaziz, Wang, Pauly: **Online Modeling For Realtime Facial Animation**, *SIGGRAPH 2013*

Conclusion

Conclusion



[Newcombe et al. 2011]

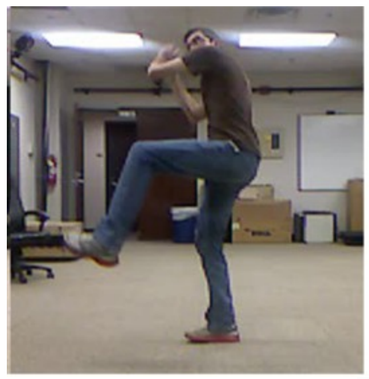


[Li et al. 2013]

Conclusion



[Newcombe et al. 2011]





[Wei et al. 2012]



[Li et al. 2013]

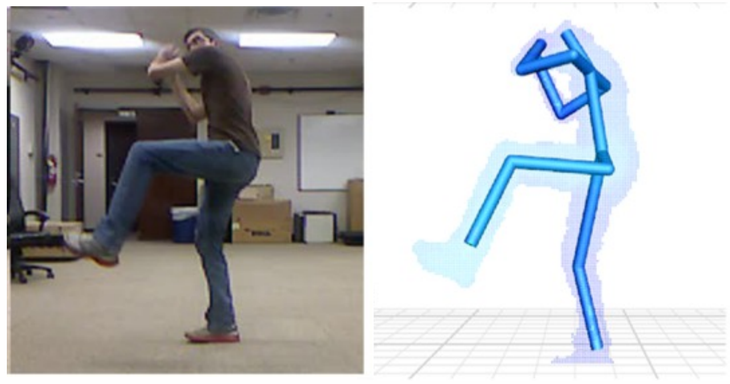


[Schroder et al. 2014]

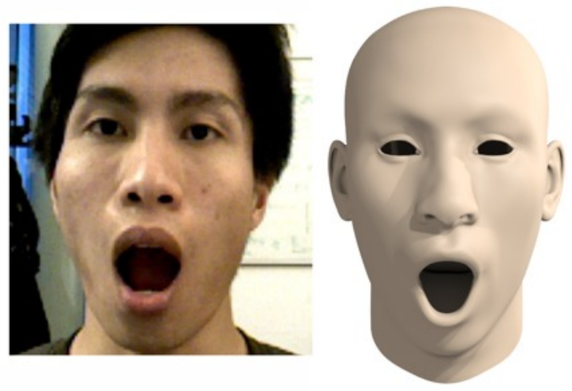
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[Bouaziz et al. 2013]



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[Schroder et al. 2014]



[Chen et al. 2013]

Thanks!

http://lgg.epfl.ch/2d3dRegistration

(source code, slides, course notes)

FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION

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